An open source ABAQUS/UEL implementation of the phase-field method to simulate brittle fracture

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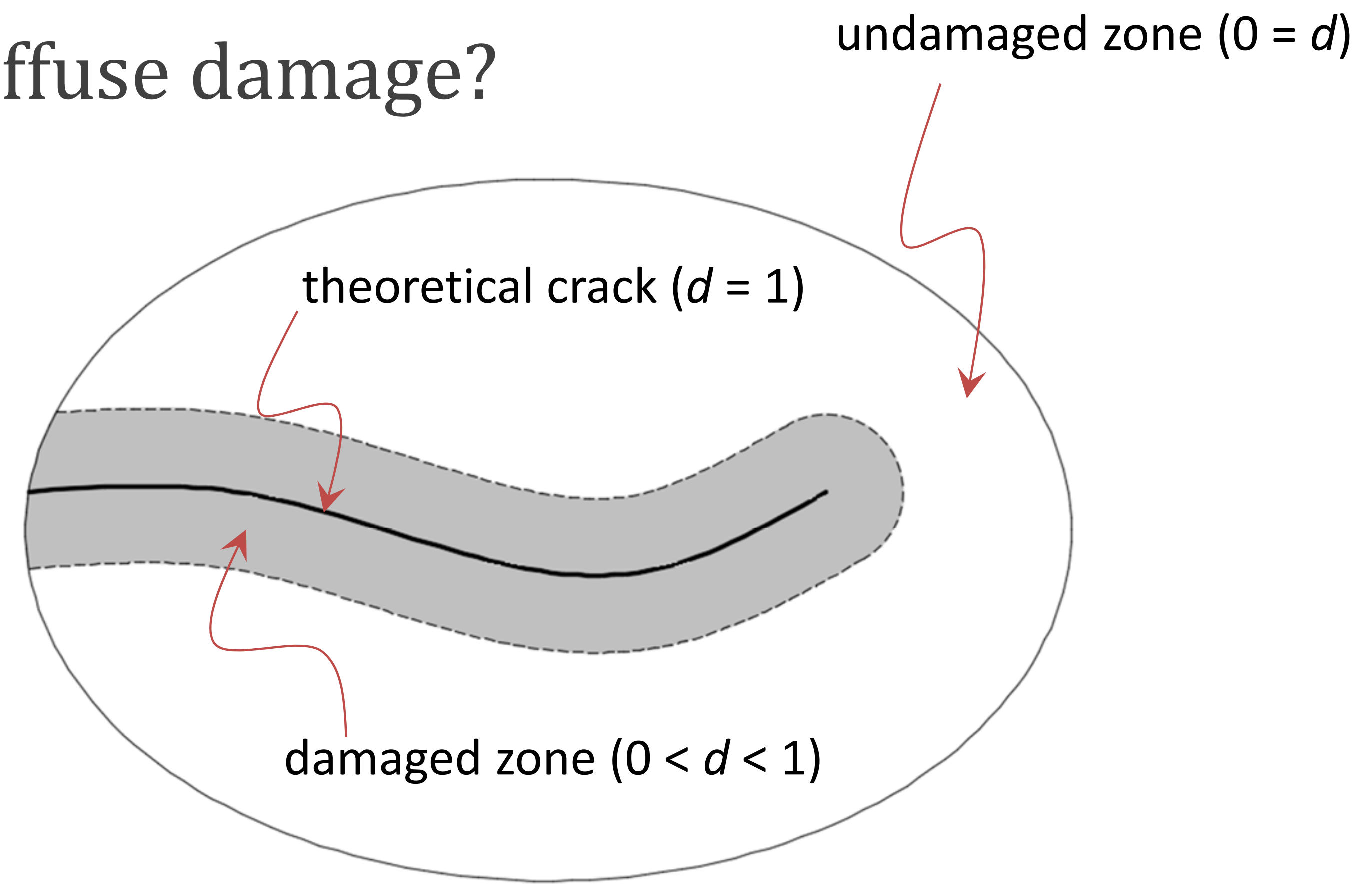
Content

Introduction

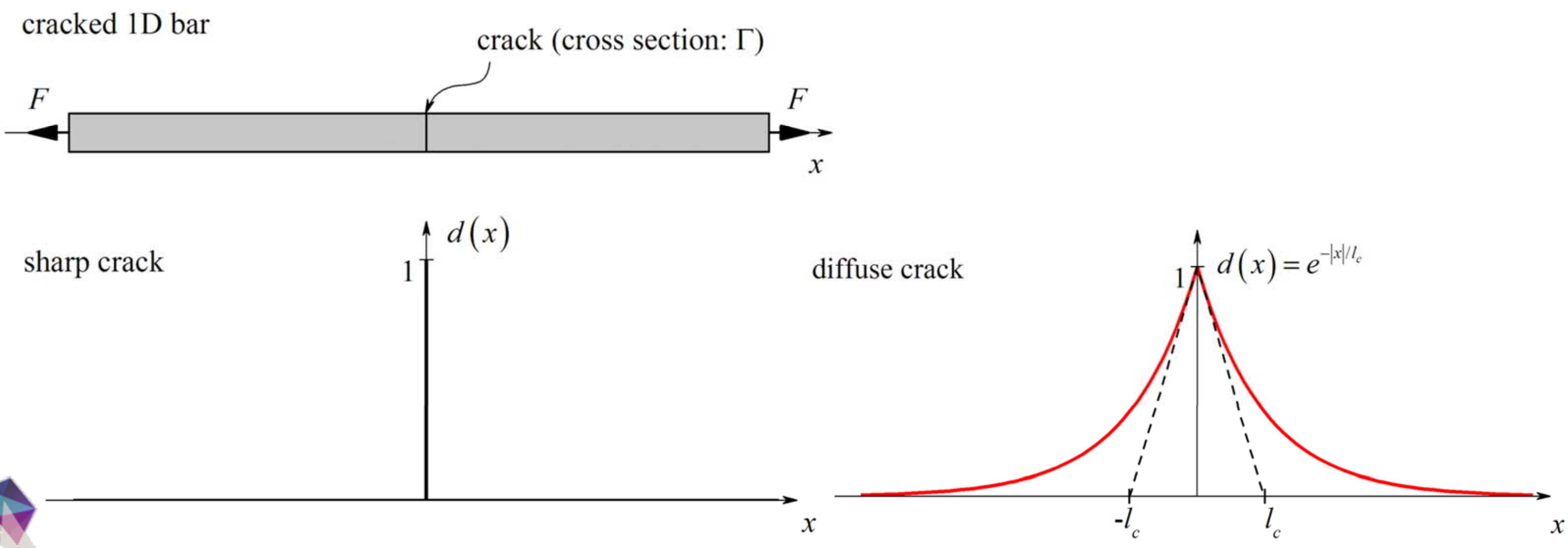
What is diffuse damage?



Solving **fracture mechanics** problem with Partial Differential Equations (**PDE**s)

Introduction

What is diffuse damage?



Solving **fracture mechanics** problem with Partial Differential Equations (**PDE**s)

Introduction

How do we approximate it with a phase-field?

1. Brittle fracture $$-\frac{\partial \psi}{\partial a} = g_c$$ Griffith, 1921

2. Minimization problem $$E(\mathbf{u},\Gamma) = \int_\Omega \psi(\varepsilon(\mathbf{u}))\,d\Omega + g_c \int_\Gamma d\Gamma$$

Mumford & Shah, 1989

Francfort & Marigo, 1998

3. Crack energy density

$$E(\mathbf{u},d) = \int_\Omega g(d)\psi(\varepsilon(\mathbf{u}))\,d\Omega + g_c \int_\Omega \left(\frac{1}{2l_c} d^2 + \frac{l_c}{2} |\nabla d|^2 \right) d\Omega$$

$l_c \to 0$ Γ converges

$\dot{d} \geq 0$

crack energy density - γ

Ambrosio & Tortorelli, 1990

Bourdin et al., 2000

Amor et al., 2009

Miehe et al., 2010a

Phase-field method

Prof. Dr.-Ing.
Christian Miehe
(1956 – †2016)

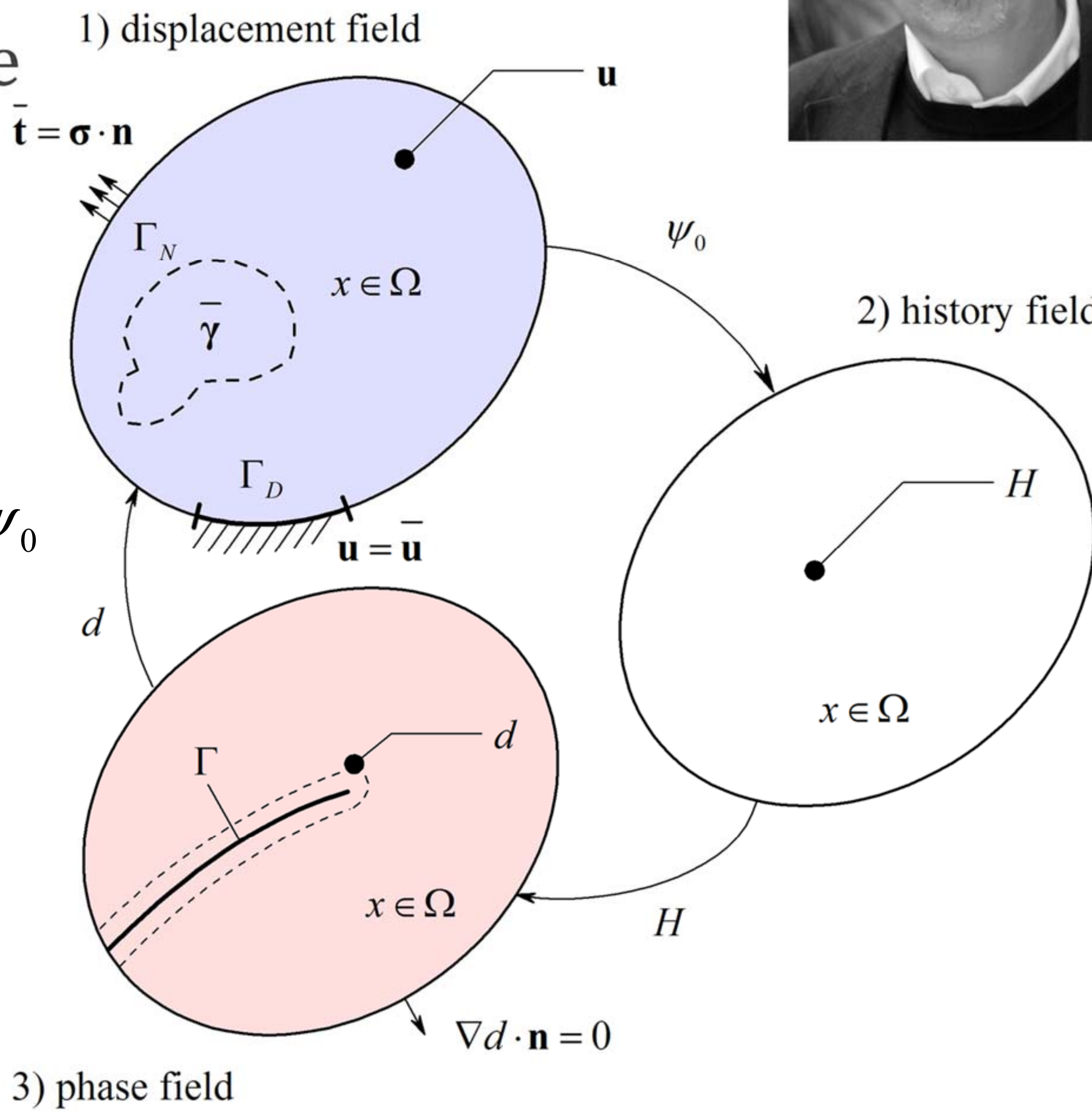
Staggered scheme

Miehe et al., 2010b

$E^u(\mathbf{u}, d)$

$E^d(d, H_n)$

$H_n = \psi_0(\varepsilon(\mathbf{u})) \quad \text{if} \quad H_{n-1} < \psi_0$



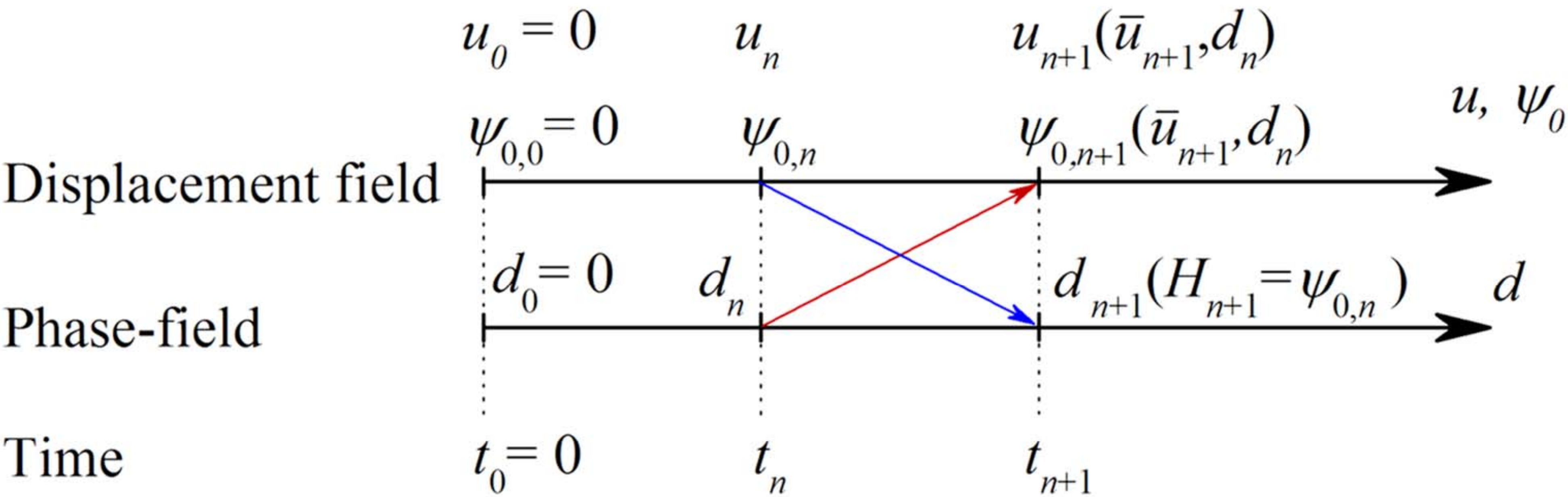
Robustness!!!

Efficiency?

Molnár & Gravouil, 2017

Phase-field method

Staggered scheme

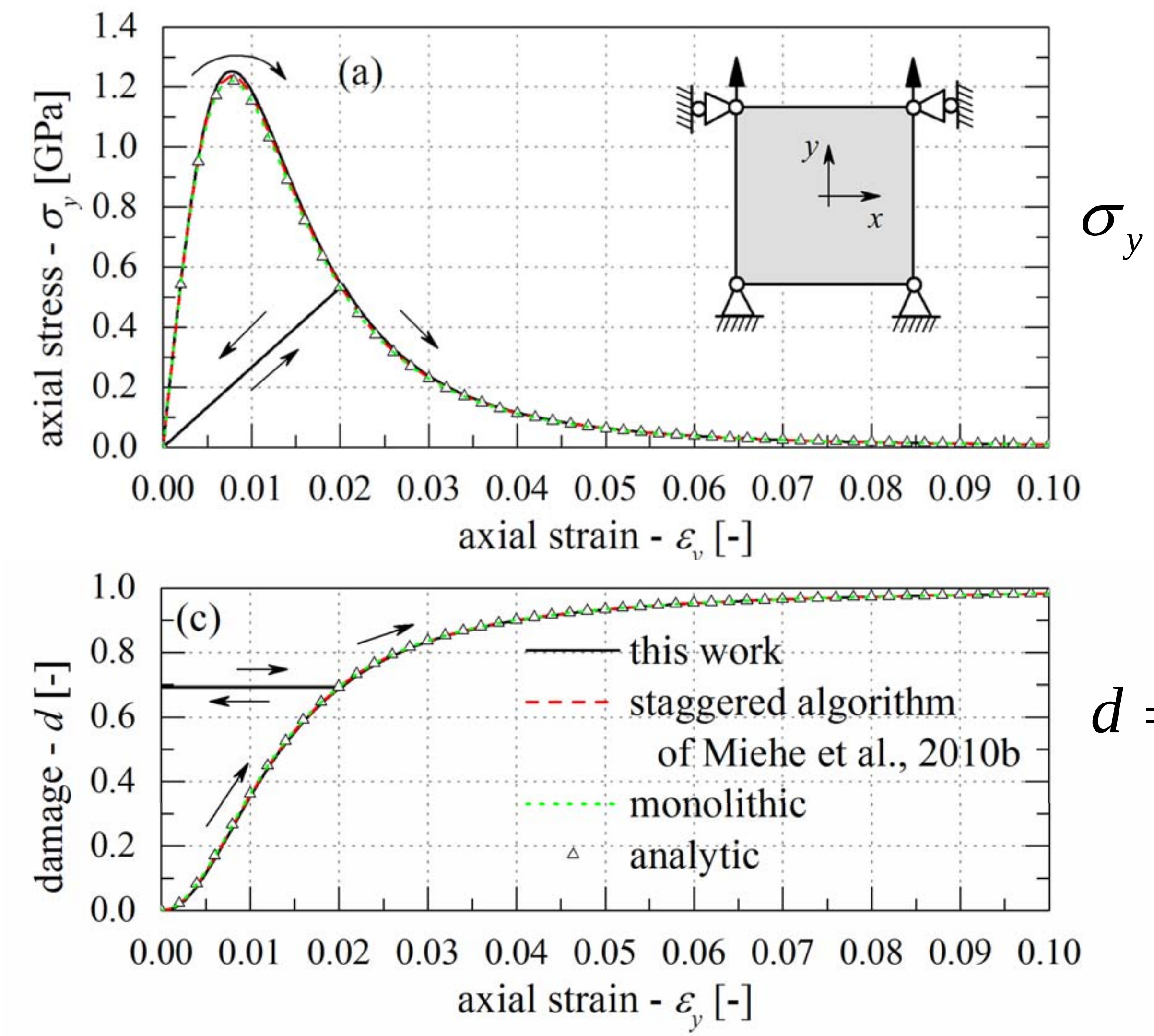


Molnár & Gravouil, 2017

Phase-field method

c_{22} - (2,2) element of the stiffness matrix

Single element solution

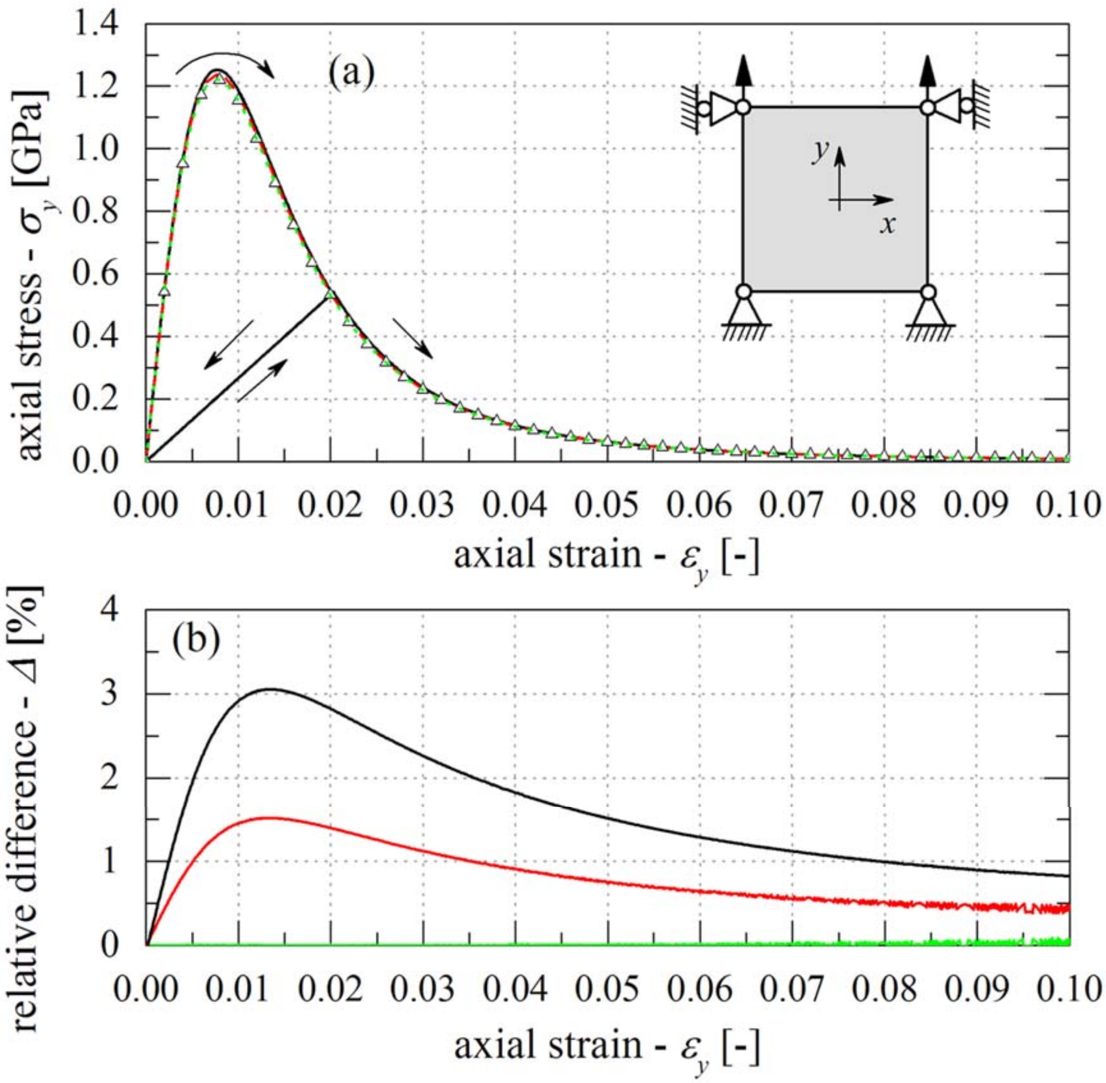


$$\sigma_y = (1-d)^2 \varepsilon_y c_{22}$$

$$d = \frac{\varepsilon_y^2 c_{22}}{\frac{g_c}{l_c} + \varepsilon_y^2 c_{22}}$$

Phase-field method

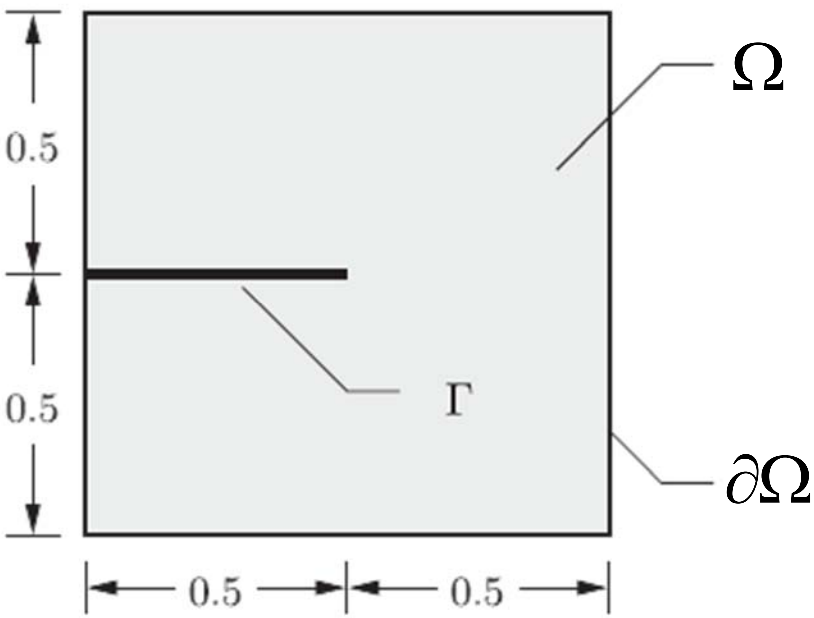
Single element solution



$$\Delta = \frac{\sigma_y - \sigma_y^{\text{analytic}}}{\sigma_y^{\text{analytic}}}$$

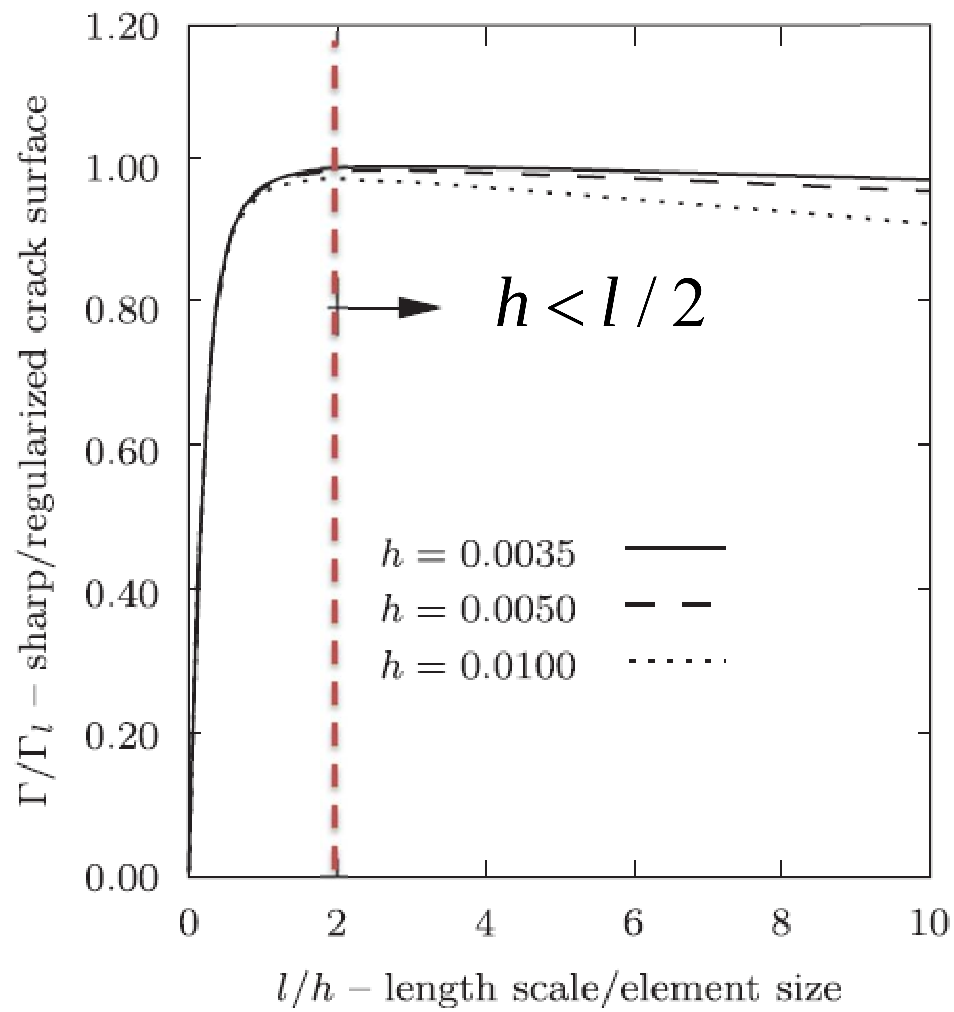
Parameters

How fine should the mesh be?



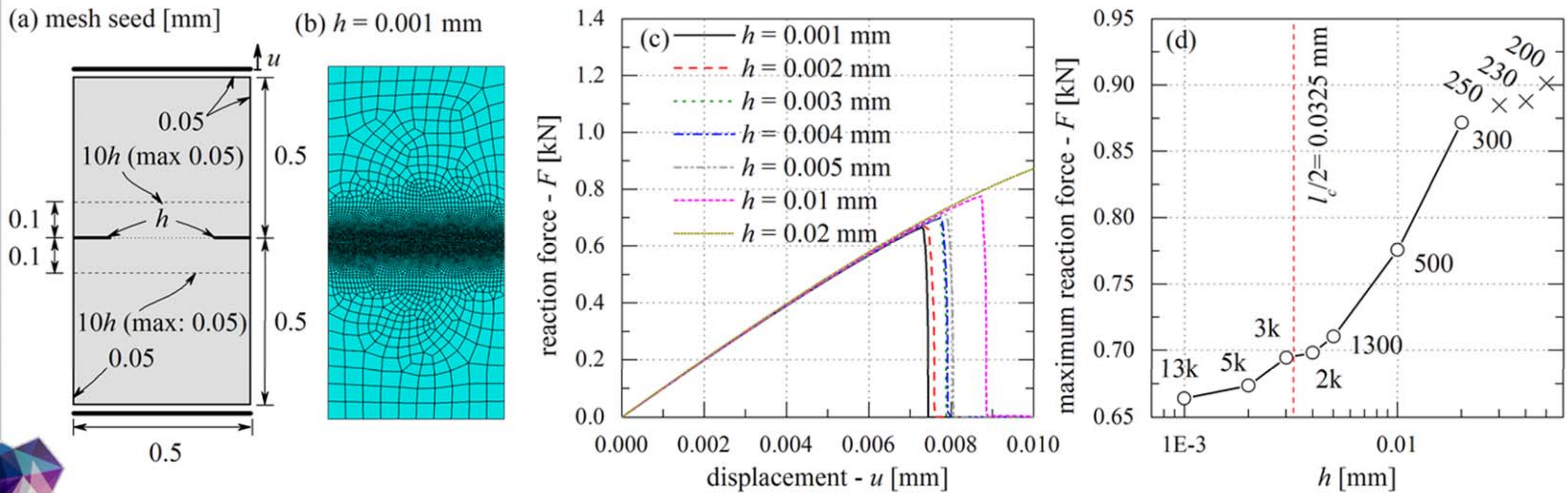
$\Gamma = 0.5$ ← **theoretical**

$$\Gamma_l = \int_\Omega \gamma \, d\Omega$$



Parameters

Double notched specimen under tension

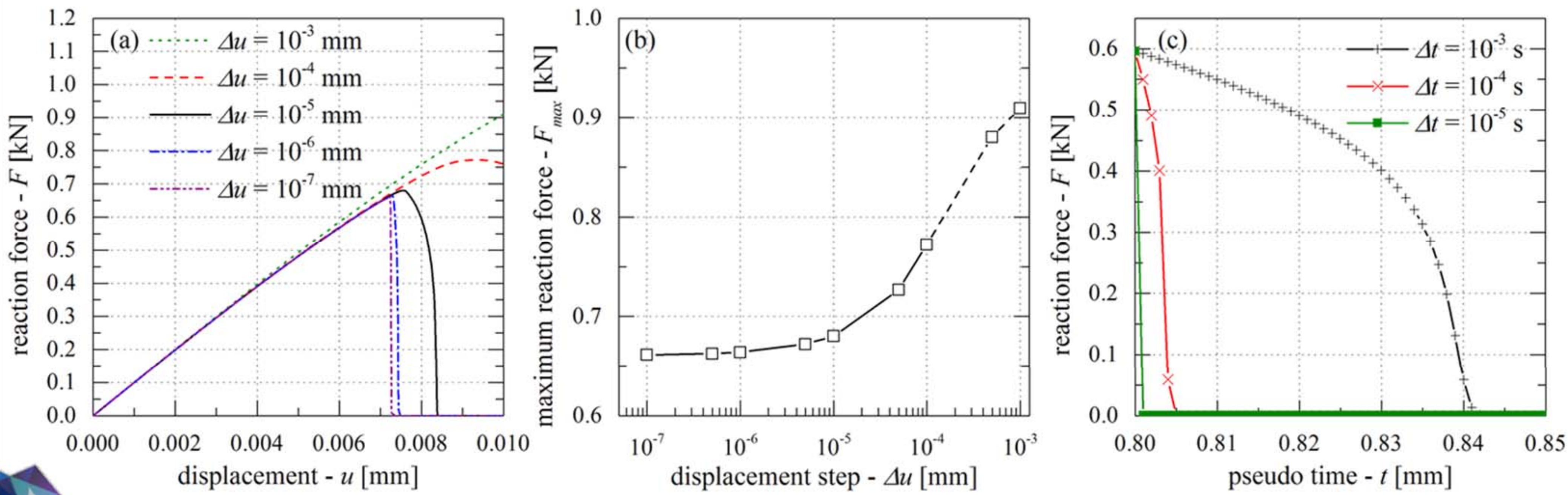


Effect of FE mesh

Molnár & Gravouil, 2017

Parameters

Double notched specimen under tension

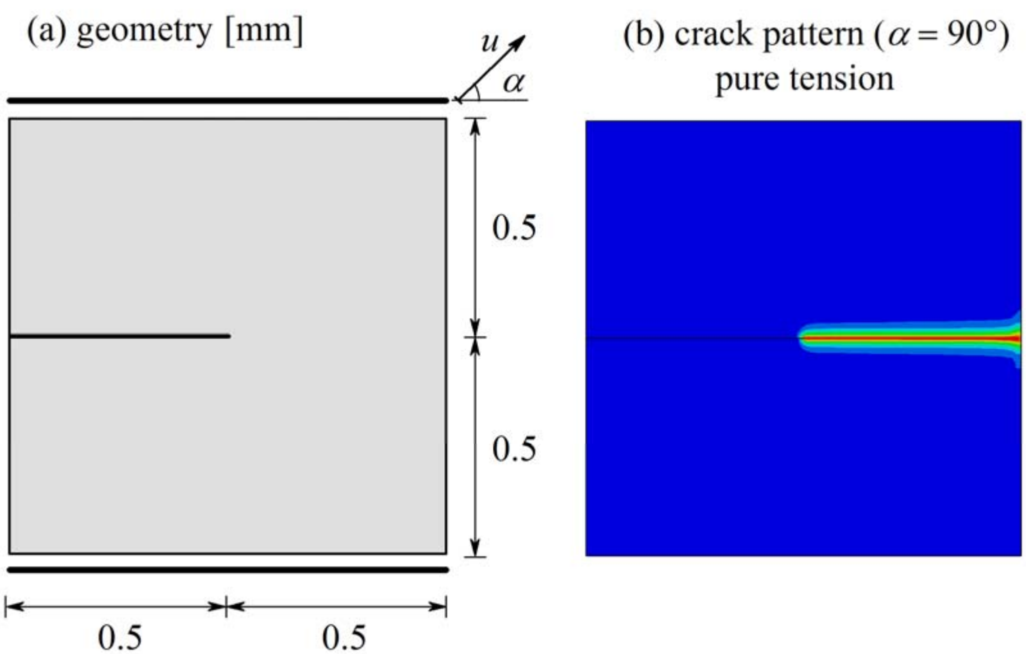


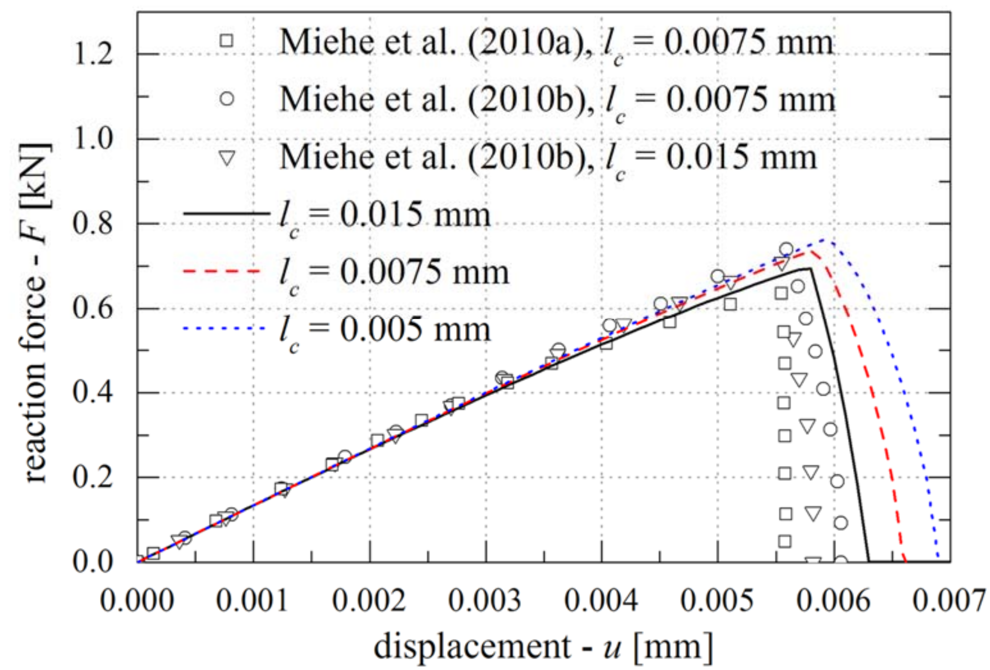
Effect of time step size

Molnár & Gravouil, 2017

Parameters

Single notched specimen under tension



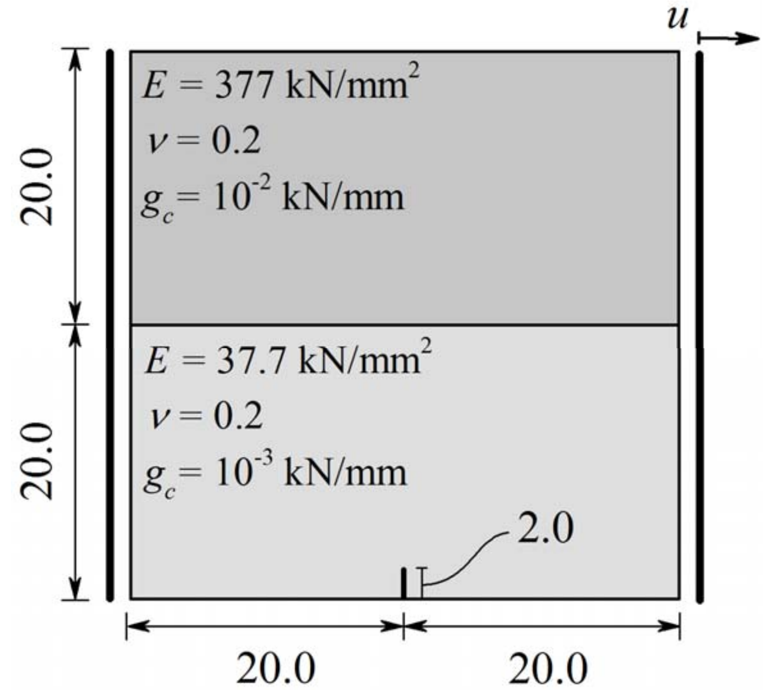


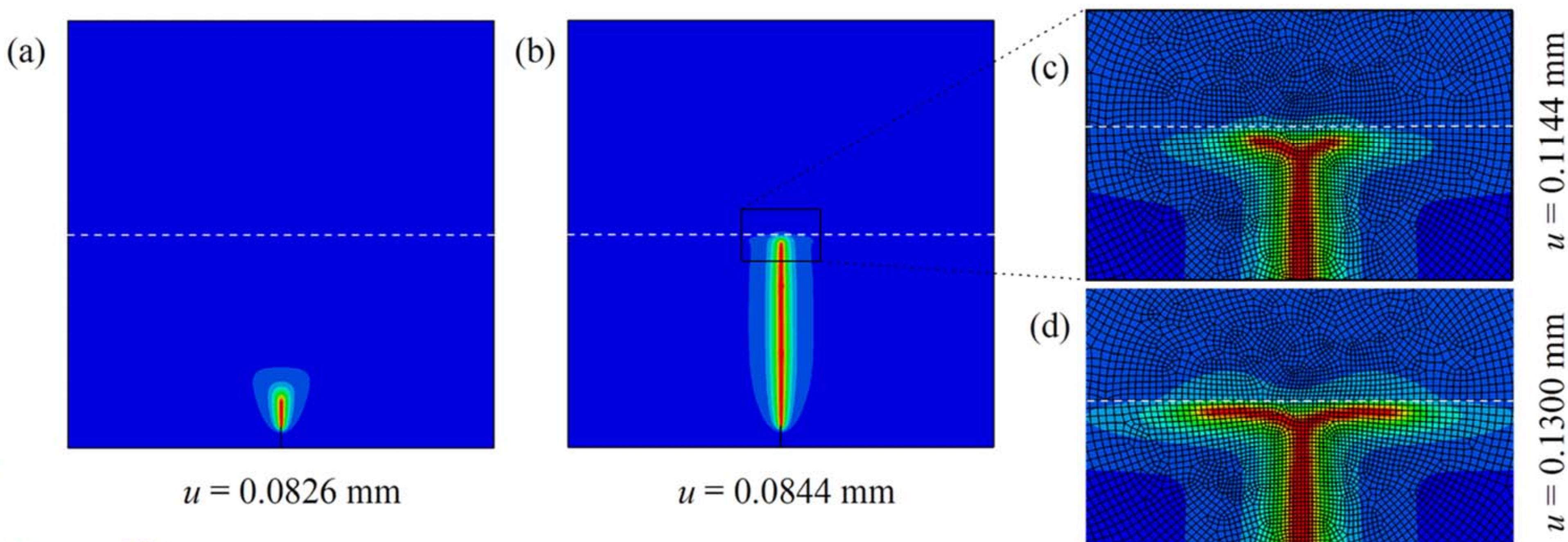
Effect of length-scale

Molnár & Gravouil, 2017

Examples

Crack branching



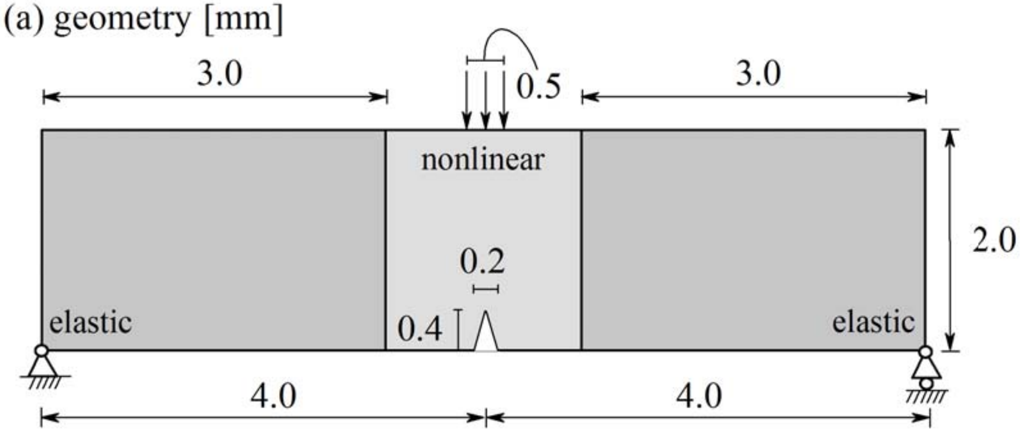


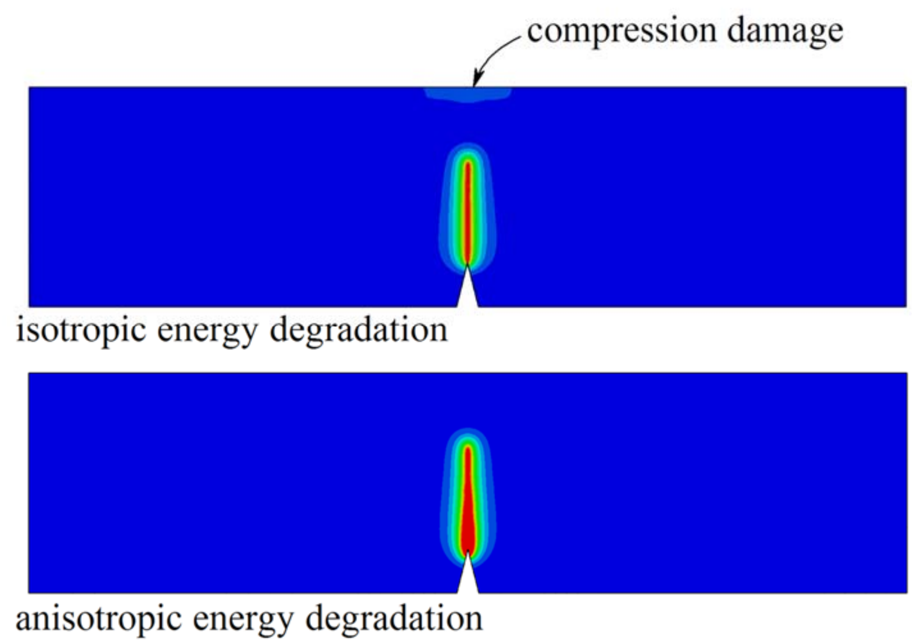
Molnár & Gravouil, 2017

Options

Positive and negative energy degradation

(a) geometry [mm]





$$\psi_0^{\pm} = \frac{E\nu}{(1+\nu)(1-2\nu)} \left\langle tr(\boldsymbol{\varepsilon}) \right\rangle_{\pm}^{2} + \frac{E}{2(1+\nu)} \left(\left\langle \varepsilon_2 \right\rangle_{\pm}^{2} + \left\langle \varepsilon_2 \right\rangle_{\pm}^{2} \right)$$

$$\psi = \psi_0^{+} g(d) + \psi_0^{-}$$

Molnár & Gravouil, 2017

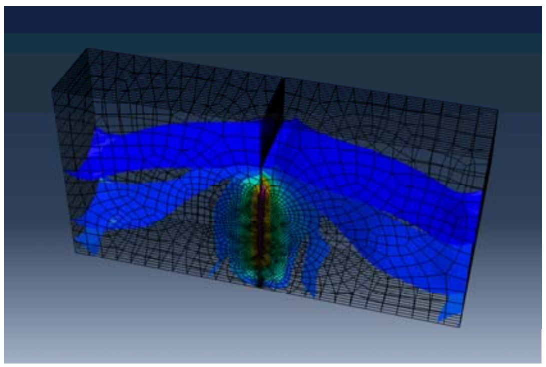
Open source implementation

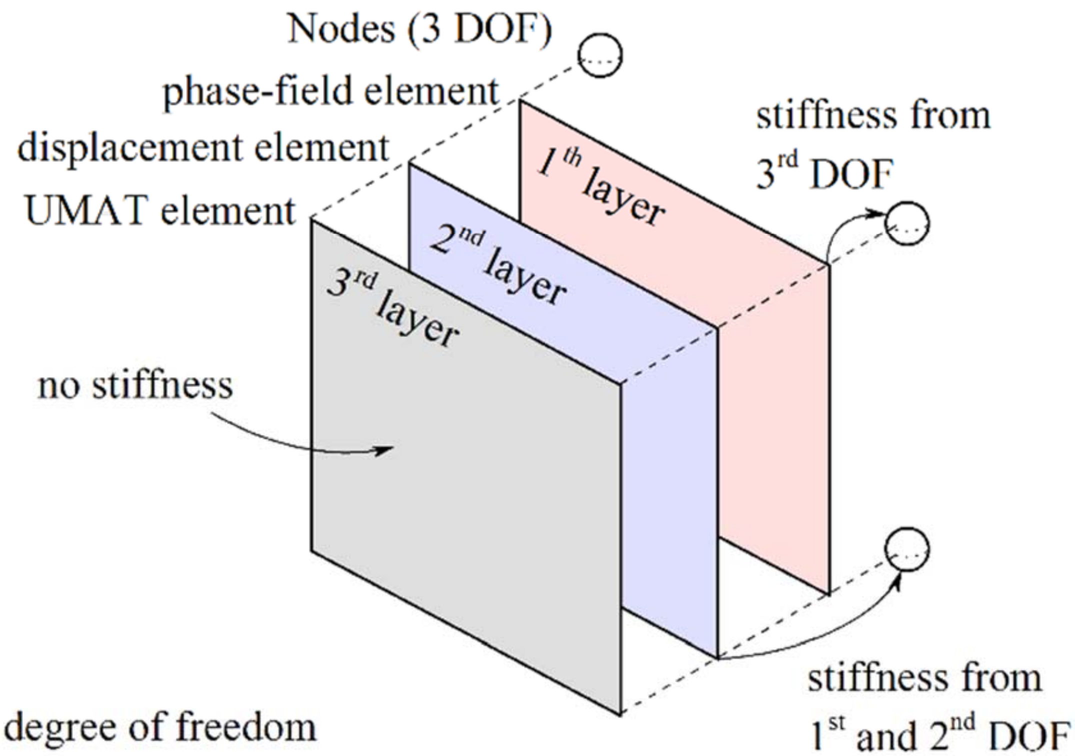
ABAQUS/UEL option (ABAQUS + FORTRAN compiler)

stiffness matrix + **residue vector** for every element

Visualization

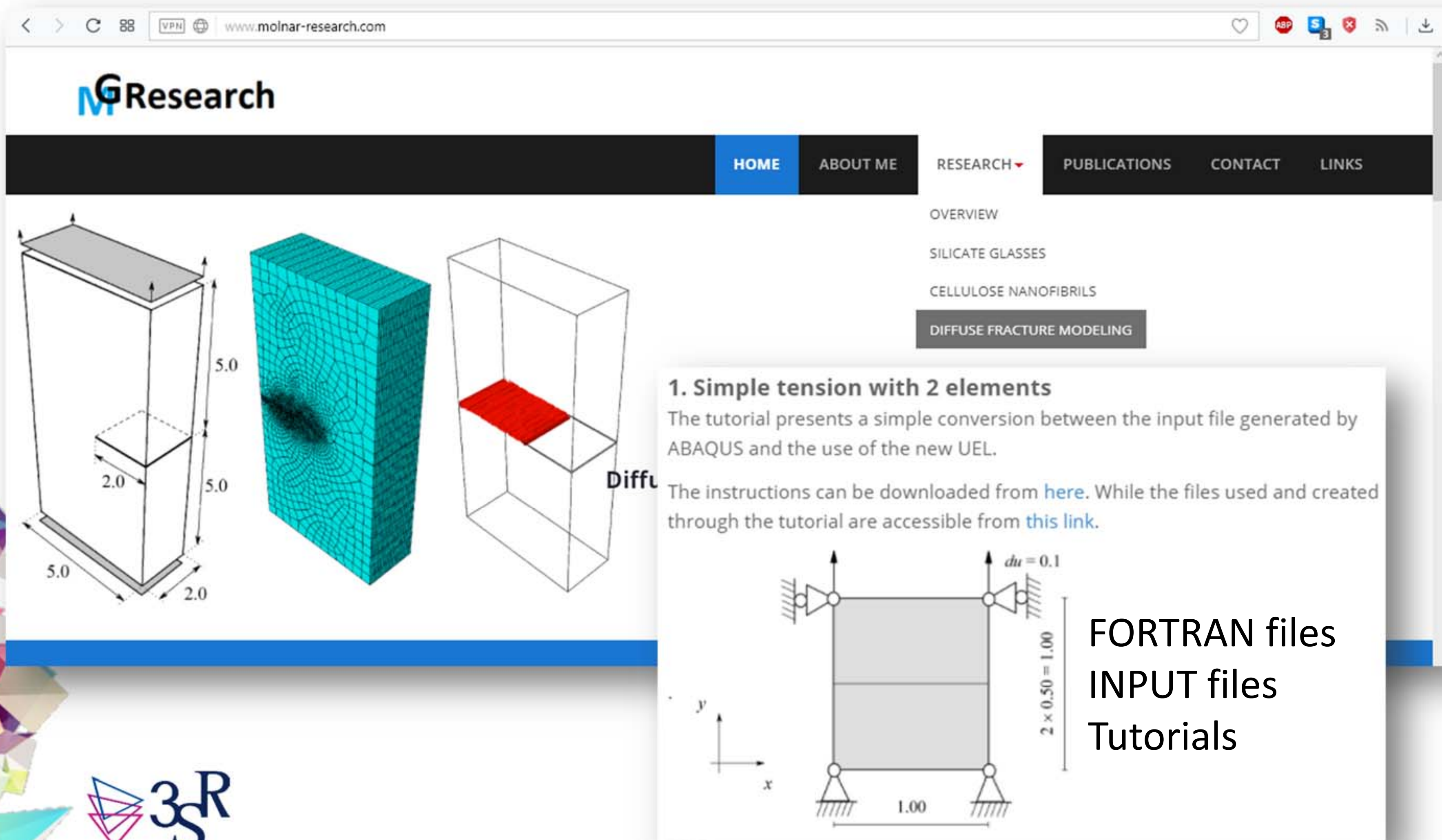
FORTRAN and ABAQUS files are available in both **2D** and **3D**



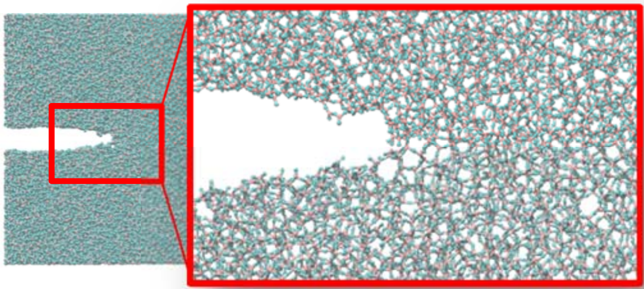


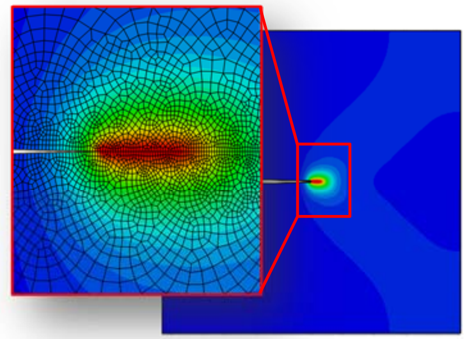
Open source implementation

Examples and tutorials: www.molnar-research.com



Conclusion





Phase-field

XFEM/GFEM

Cohesive Zones

Predefined crack

Advantages and disadvantages

- crack initiation, propagation
- branching, merging
- fixed mesh
- fully 3D

- fine mesh
- finite crack size
- efficiency/robustness

Versatility

dynamics, shells, nonlinear elasticity, large strains, coupled problems, plasticity, anisotropy, etc…

References

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Thank you for your attention

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