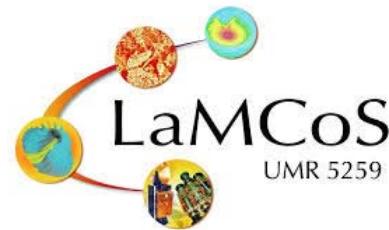


An open source ABAQUS/UEL implementation of the phase-field method to simulate brittle fracture

Gergely Molnár and Anthony Gravouil



Content

Introduction to phase-fields

1. Phase-field concept
2. Strain energy degradation
3. Staggered scheme

Parameters

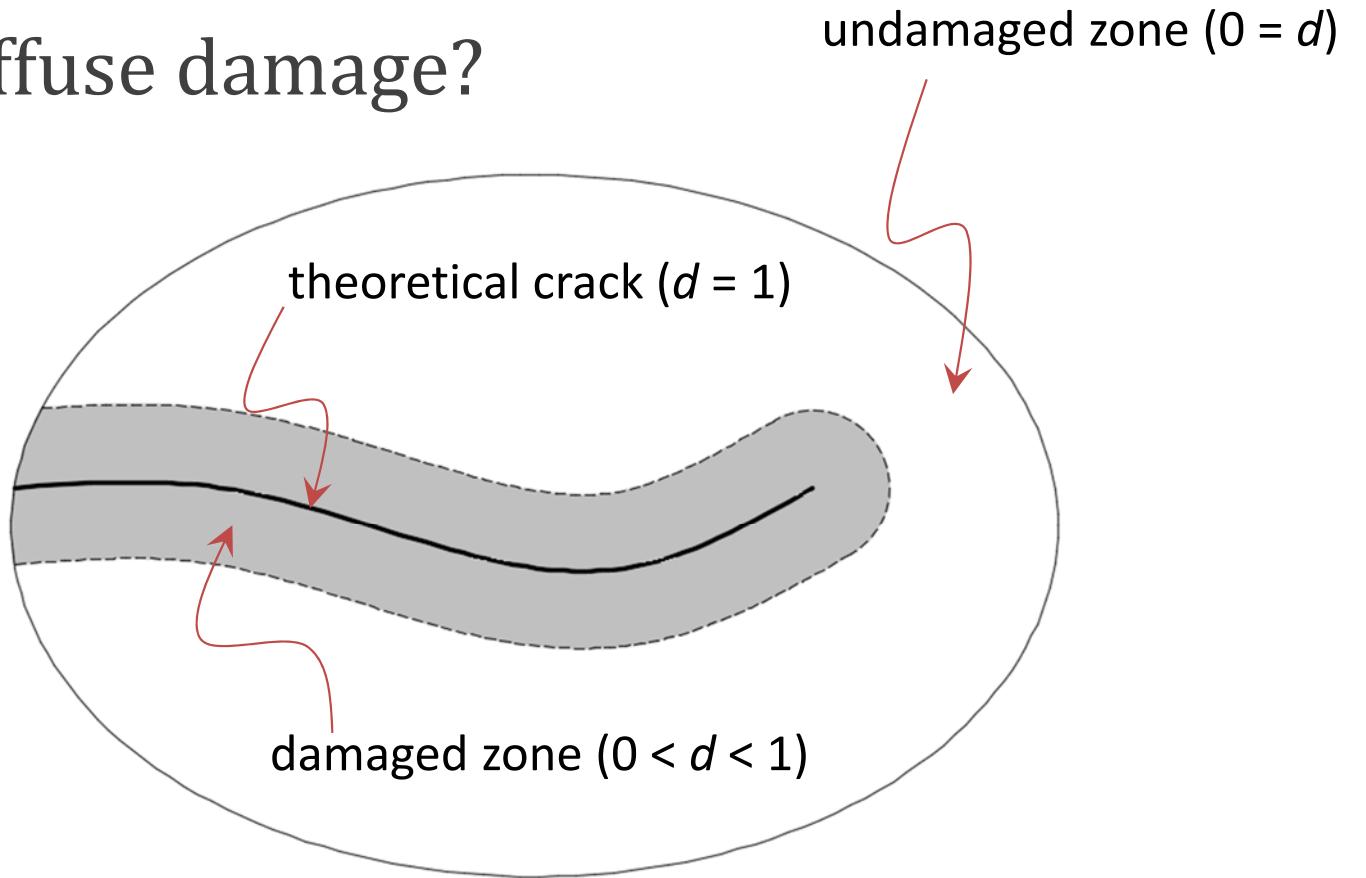
1. Finite element mesh
2. Time step
3. Length-scale parameter

ABAQUS/UEL implementation

Conclusion

Introduction

What is diffuse damage?

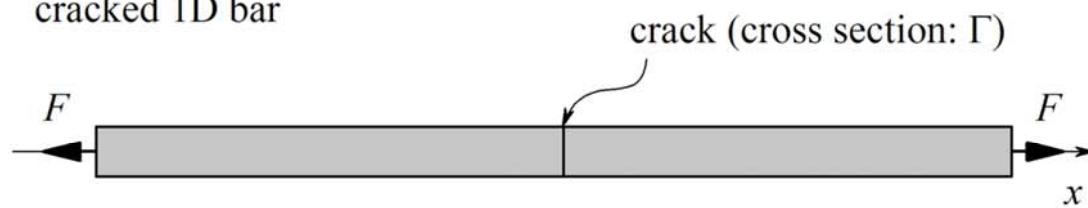


Solving **fracture mechanics** problem with
Partial Differential Equations (**PDEs**)

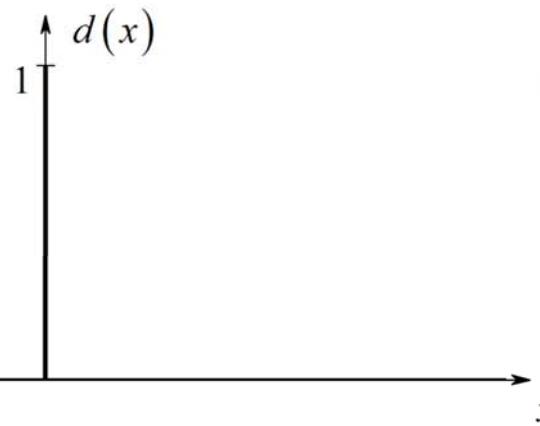
Introduction

What is diffuse damage?

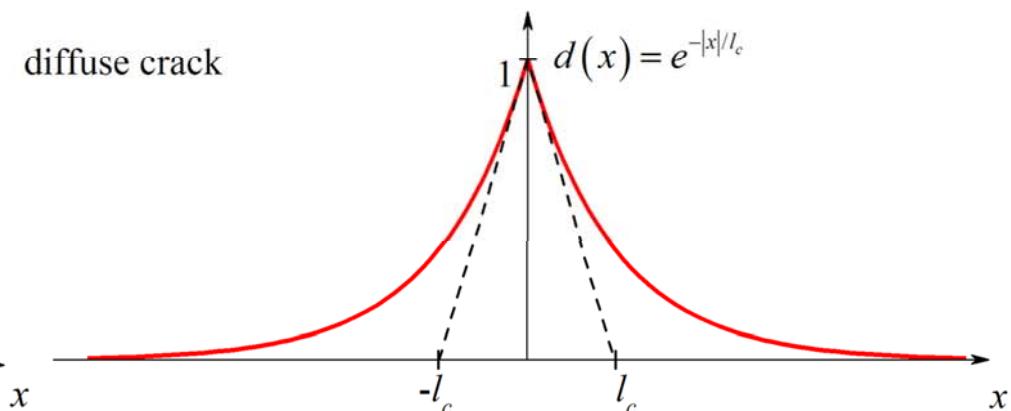
cracked 1D bar



sharp crack



diffuse crack



Solving **fracture mechanics** problem with
Partial Differential Equations (**PDEs**)

Introduction

How do we approximate it with a phase-field?

1. Brittle fracture

$$-\frac{\partial \psi}{\partial a} = g_c$$

Griffith, 1921

2. Minimization problem

$$E(\mathbf{u}, \Gamma) = \int_{\Omega} \psi(\varepsilon(\mathbf{u})) d\Omega + g_c \int_{\Gamma} d\Gamma$$

Mumford & Shah, 1989

Francfort & Marigo, 1998

3. Crack energy density

$$E(\mathbf{u}, \mathbf{d}) = \int_{\Omega} g(\mathbf{d}) \psi(\varepsilon(\mathbf{u})) d\Omega + g_c \int_{\Omega} \left(\frac{1}{2l_c} \mathbf{d}^2 + \frac{l_c}{2} |\nabla \mathbf{d}|^2 \right) d\Omega$$

$l_c \rightarrow 0$ Γ converges

$\dot{d} \geq 0$

crack energy density - γ



Ambrosio & Tortorelli, 1990

Bourdin et al., 2000

Amor et al., 2009

Miehe et al., 2010a

Prof. Dr.-Ing.
Christian Miehe
(1956 – †2016)



Phase-field method

Staggered scheme

Miehe et al., 2010b

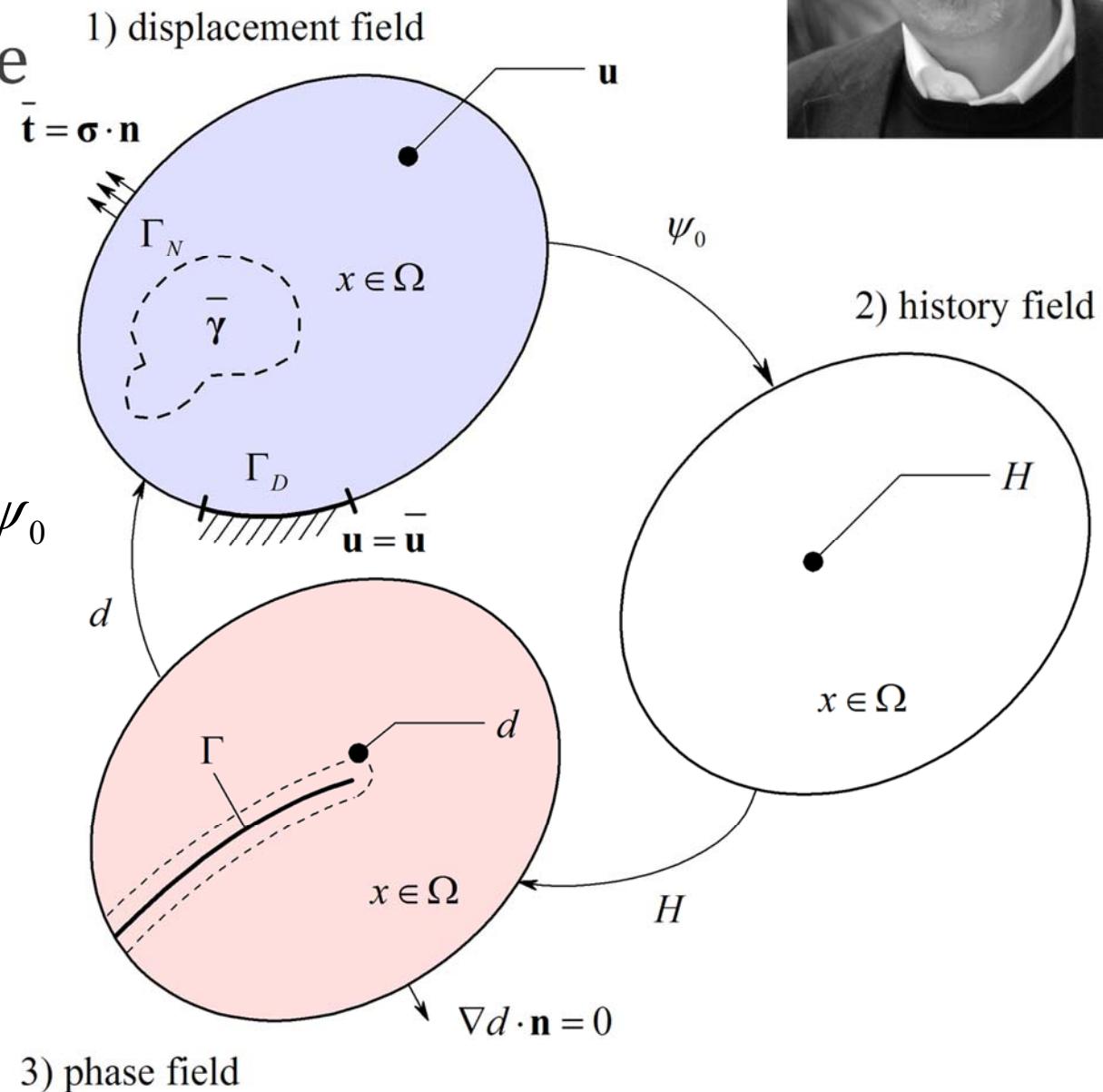
$$E^u(\mathbf{u}, d)$$

$$E^d(d, H_n)$$

$$H_n = \psi_0(\varepsilon(\mathbf{u})) \quad \text{if} \quad H_{n-1} < \psi_0$$

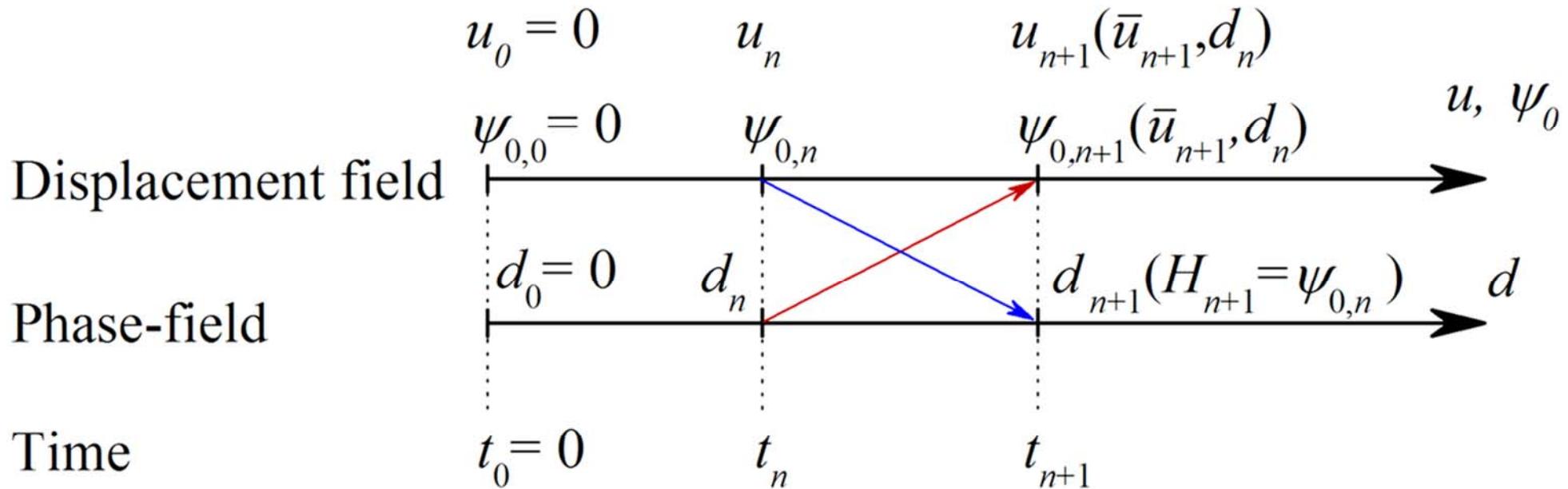
Robustness!!!

Efficiency?



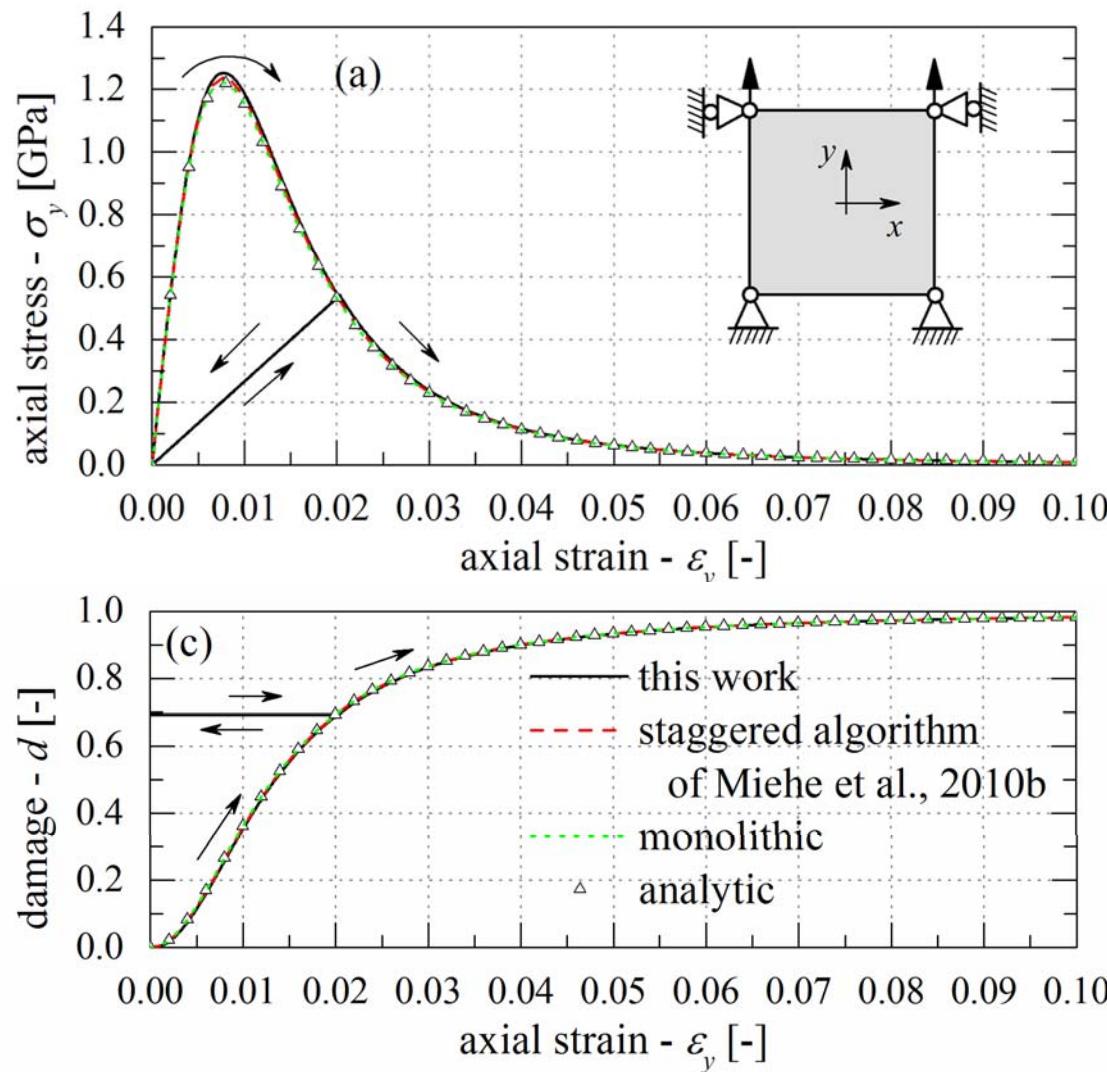
Phase-field method

Staggered scheme



Phase-field method c_{22} - (2,2) element of the stiffness matrix

Single element solution

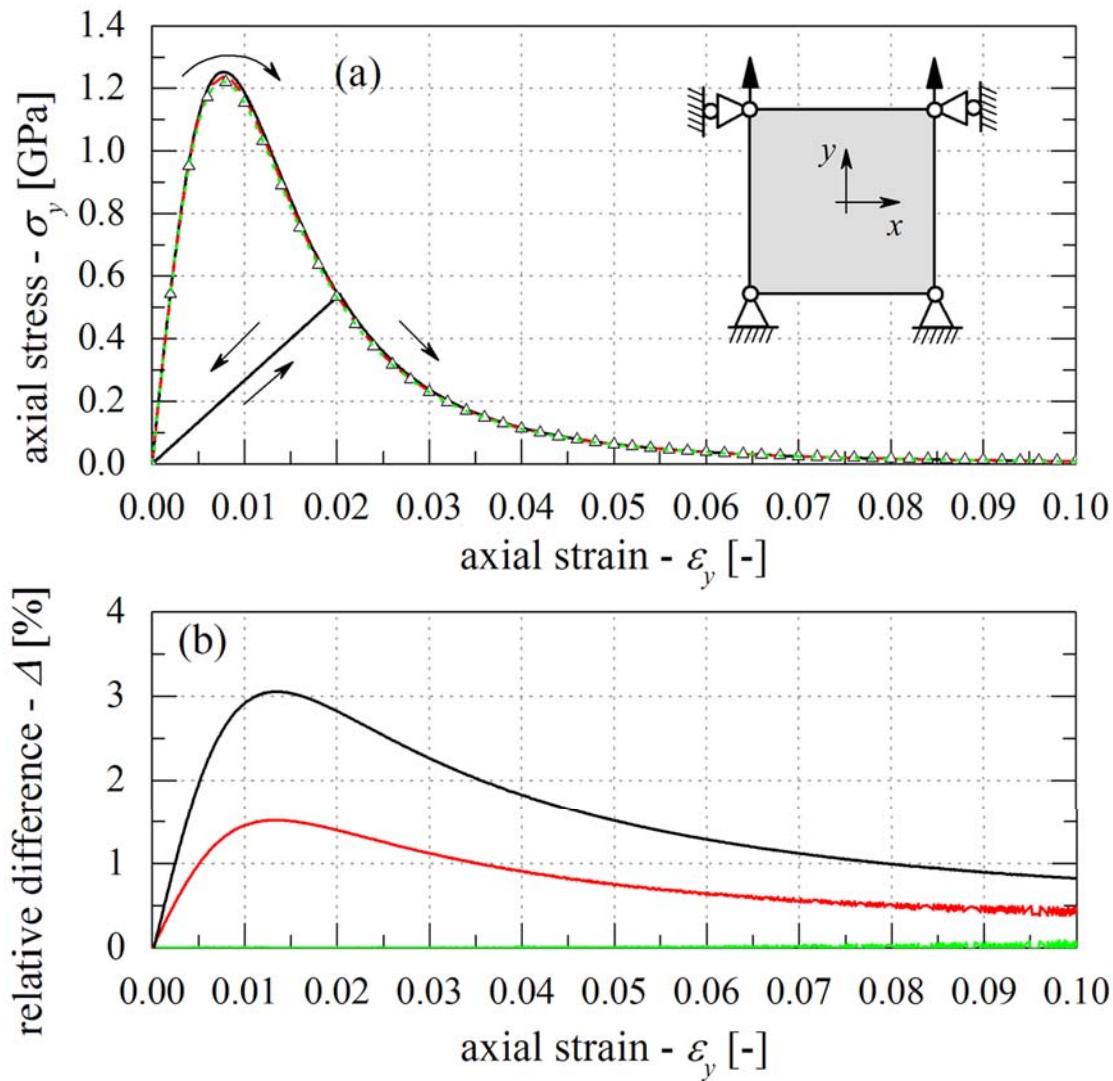


$$\sigma_y = (1-d)^2 \varepsilon_y c_{22}$$

$$d = \frac{\varepsilon_y^2 c_{22}}{\frac{g_c}{l_c} + \varepsilon_y^2 c_{22}}$$

Phase-field method

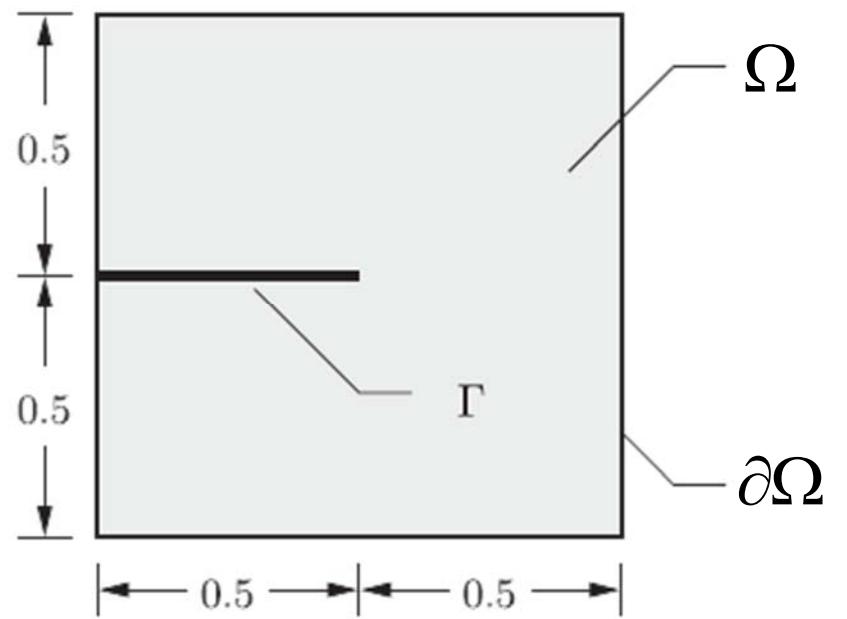
Single element solution



$$\Delta = \frac{\sigma_y - \sigma_y^{\text{analytic}}}{\sigma_y^{\text{analytic}}}$$

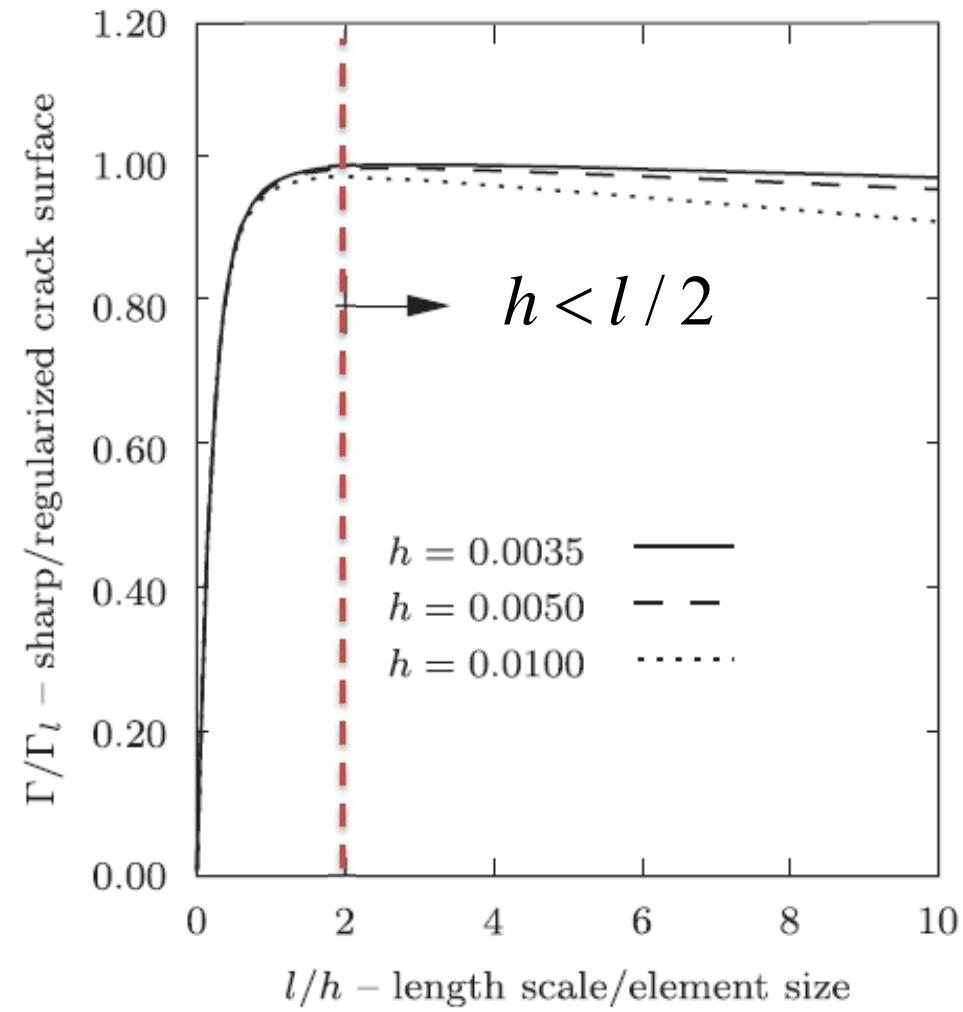
Parameters

How fine should the mesh be?



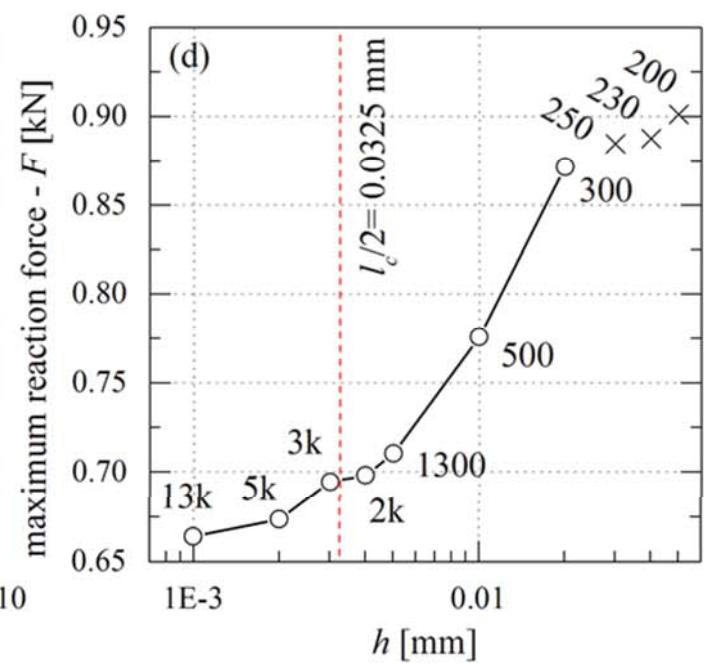
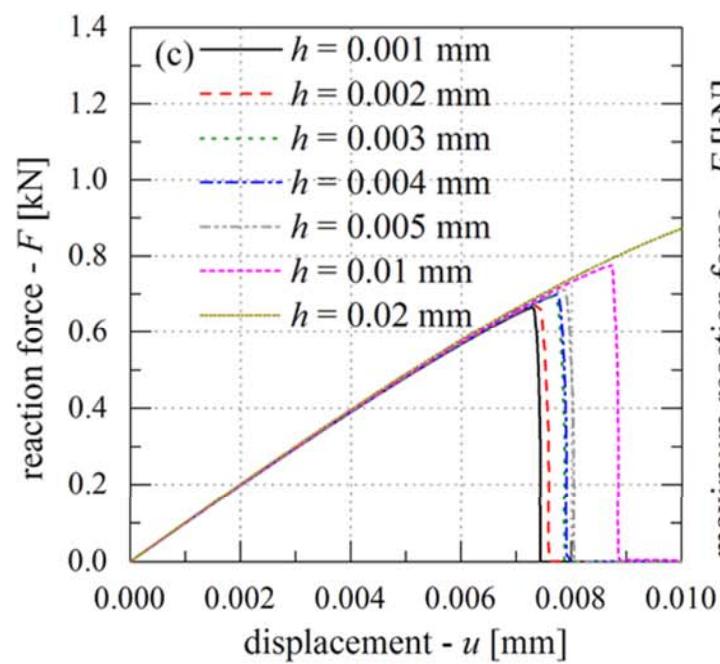
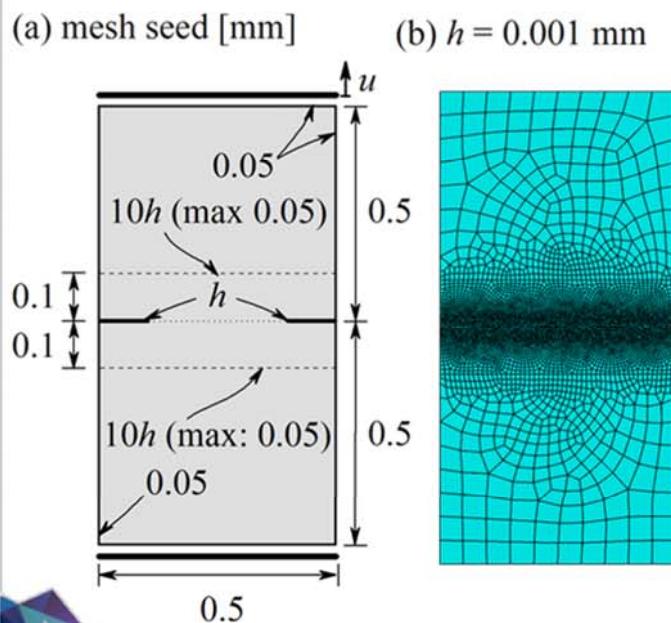
$$\Gamma = 0.5 \quad \text{← theoretical}$$

$$\Gamma_l = \int_{\Omega} \gamma d\Omega$$



Parameters

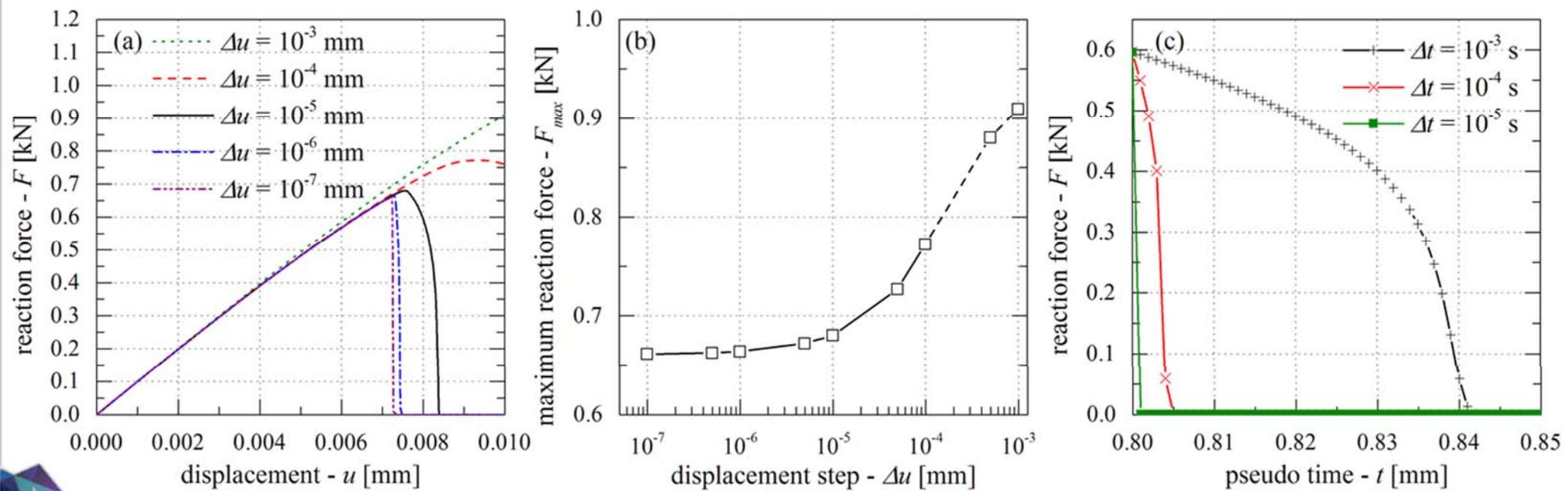
Double notched specimen under tension



Effect of FE mesh

Parameters

Double notched specimen under tension

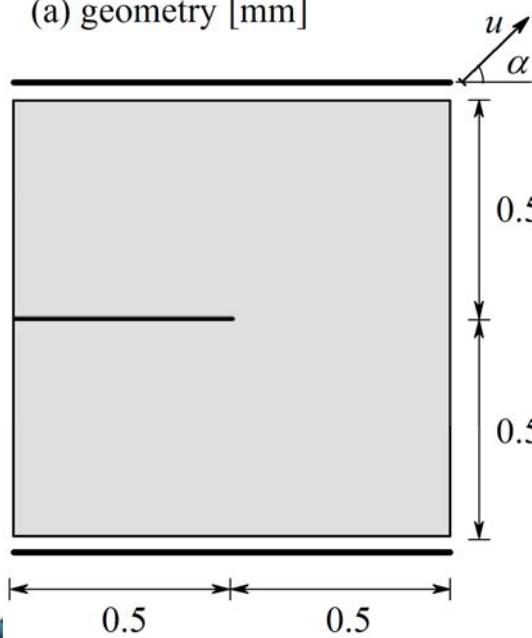
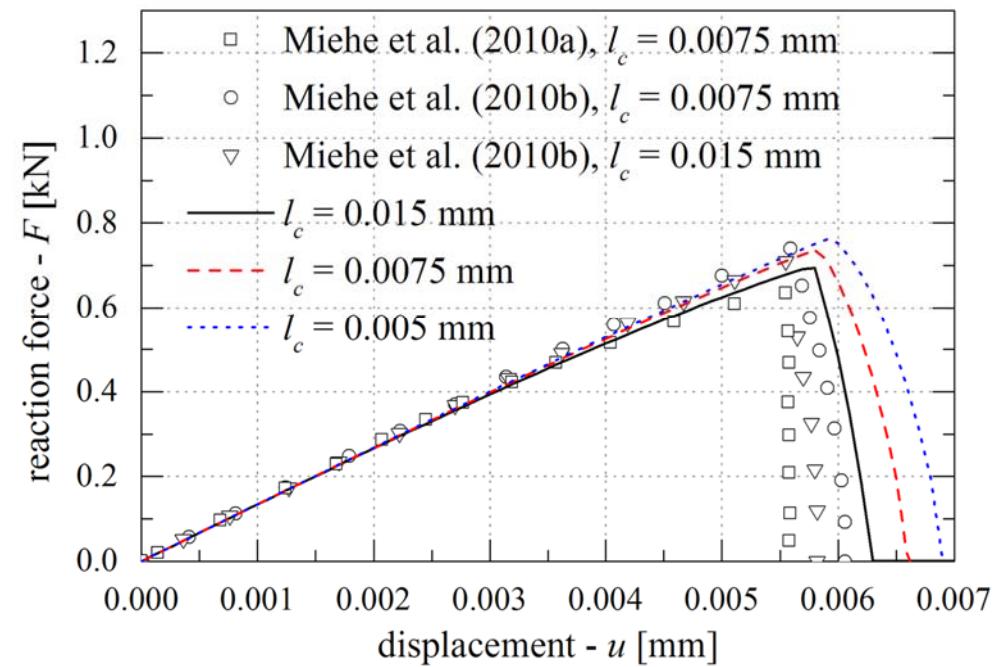
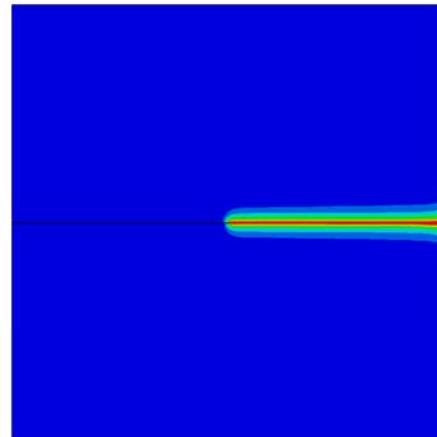


Effect of time step size

Parameters

Single notched specimen under tension

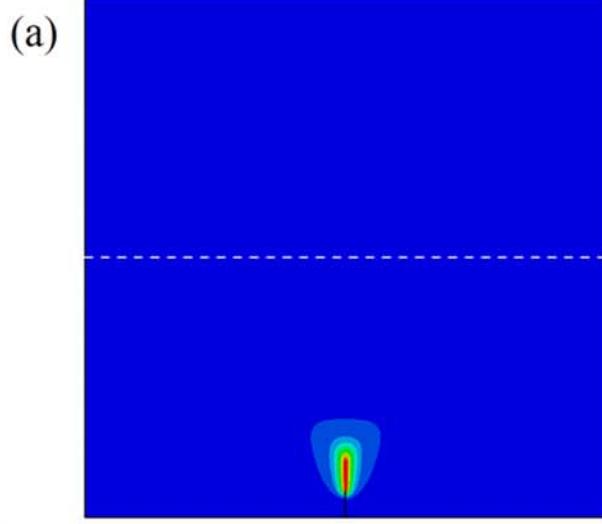
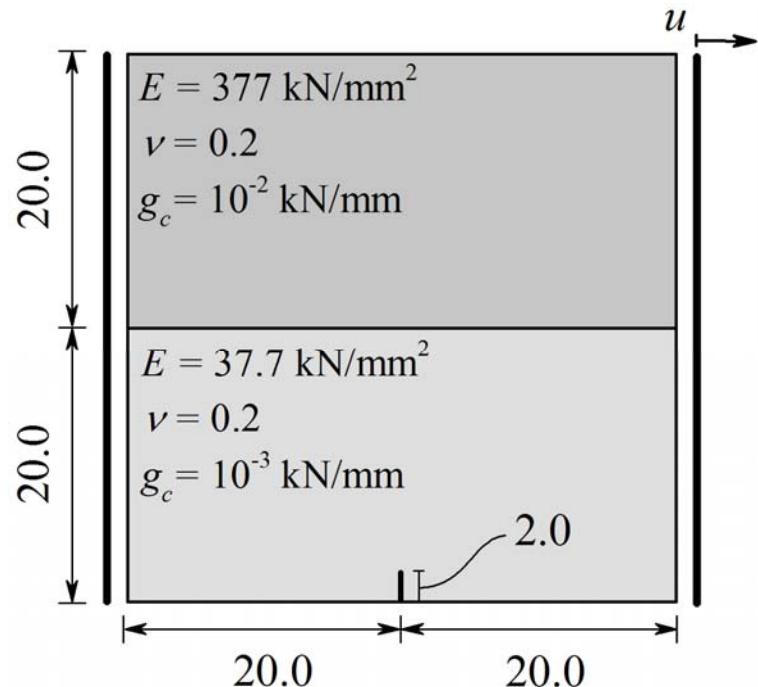
(a) geometry [mm]

(b) crack pattern ($\alpha = 90^\circ$)
pure tension

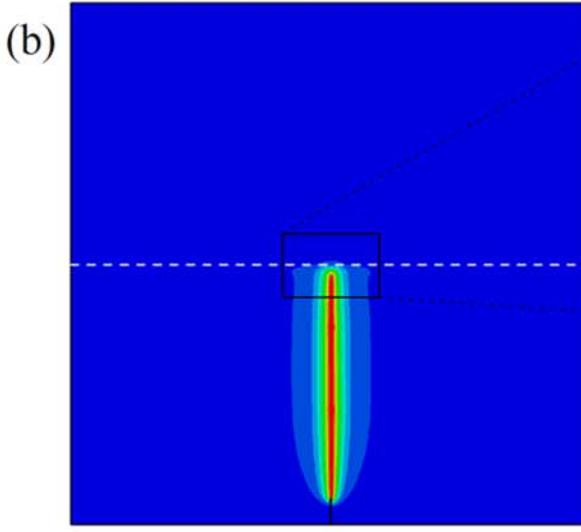
Effect of length-scale

Examples

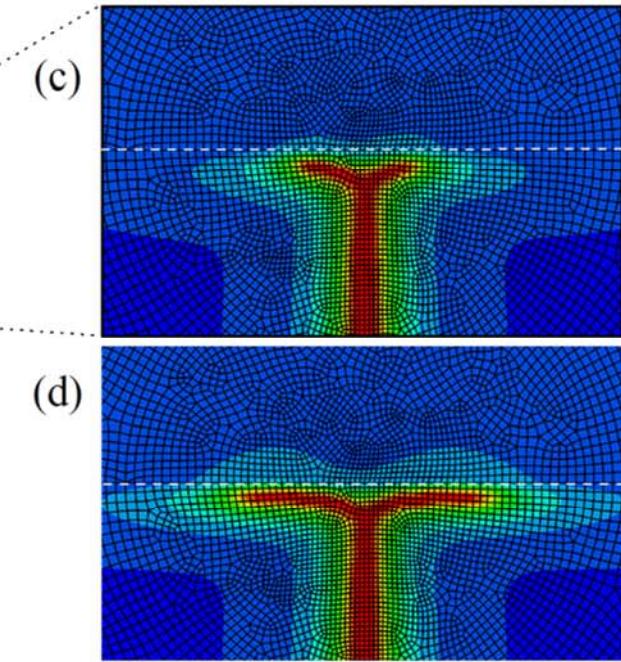
Crack branching



$u = 0.0826 \text{ mm}$



$u = 0.0844 \text{ mm}$



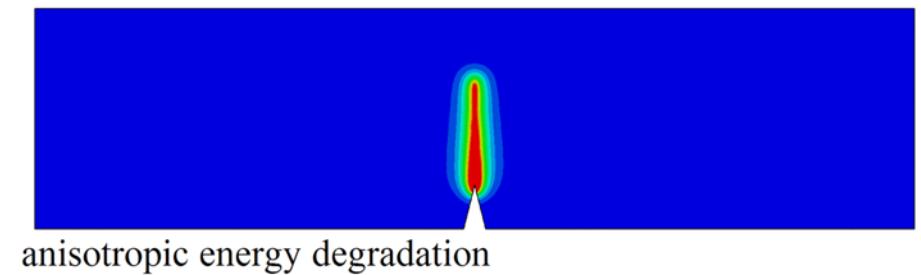
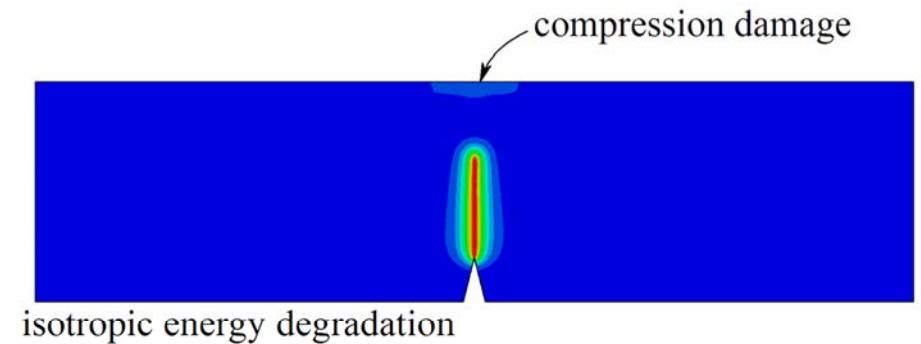
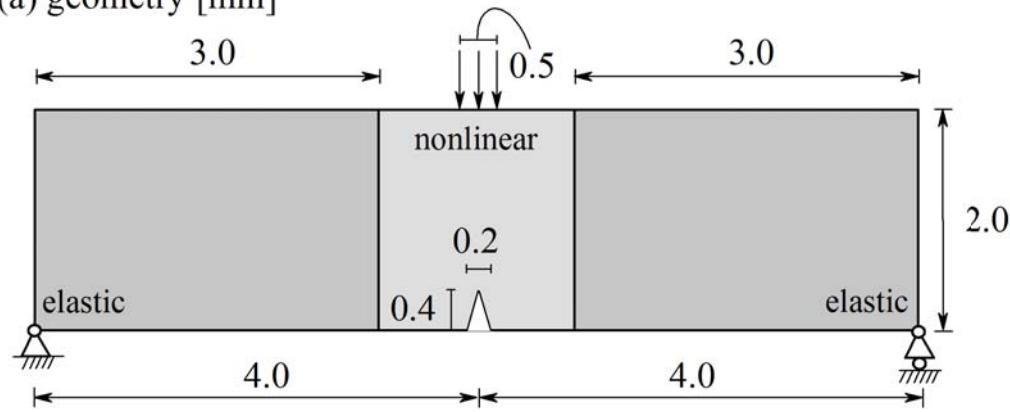
$u = 0.1144 \text{ mm}$

$u = 0.1300 \text{ mm}$

Options

Positive and negative energy degradation

(a) geometry [mm]



$$\begin{aligned} \psi_0^\pm = & \frac{E\nu}{(1+\nu)(1-2\nu)} \left\langle \text{tr}(\boldsymbol{\varepsilon}) \right\rangle_\pm^2 + \\ & + \frac{E}{2(1+\nu)} \left(\left\langle \boldsymbol{\varepsilon}_2 \right\rangle_\pm^2 + \left\langle \boldsymbol{\varepsilon}_2 \right\rangle_\pm^2 \right) \end{aligned}$$

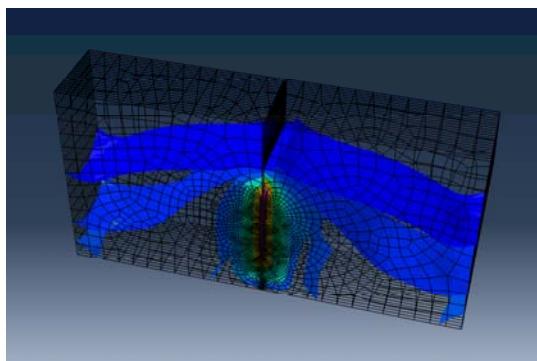
$$\psi = \psi_0^+ g(d) + \psi_0^-$$

Open source implementation

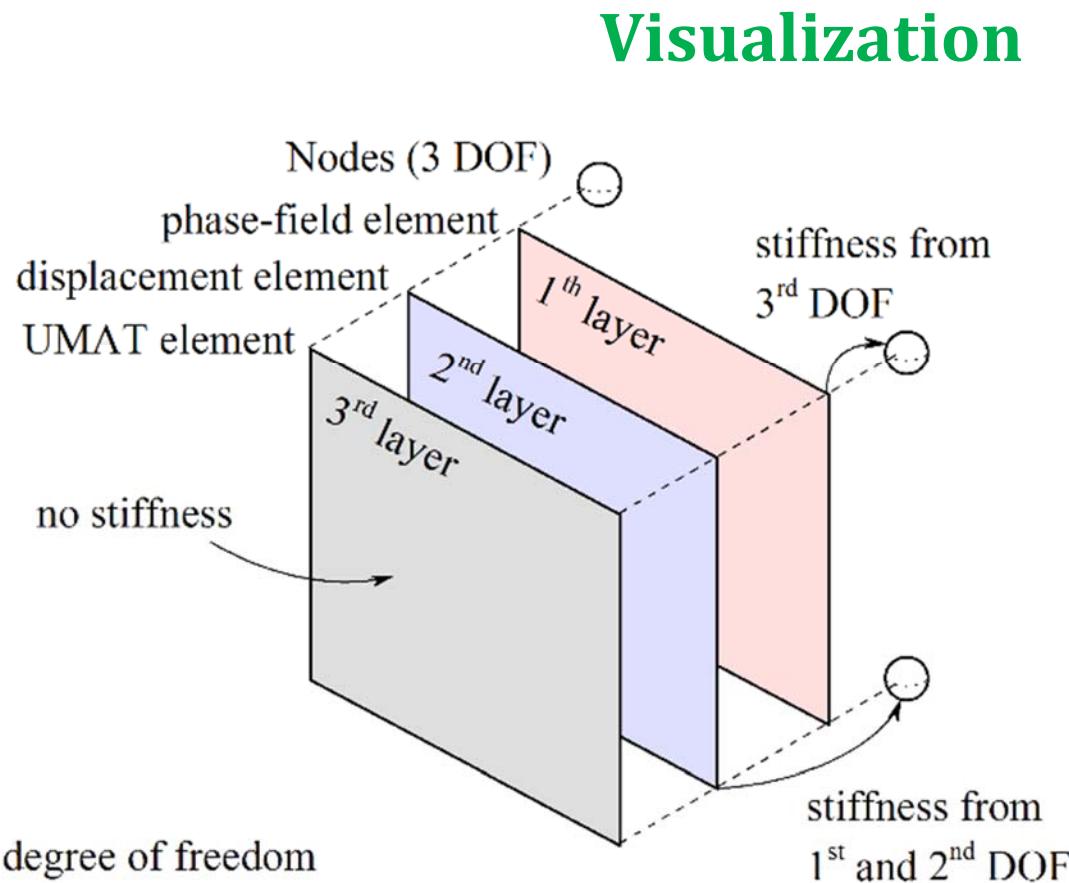
ABAQUS/UEL option (ABAQUS + FORTRAN compiler)

stiffness matrix + **residue vector** for every element

FORTRAN and ABAQUS
files are available in
both **2D** and **3D**



DOF - degree of freedom



Open source implementation

Examples and tutorials: www.molnar-research.com

The screenshot shows a web browser displaying the website www.molnar-research.com. The page features a navigation bar with links for HOME, ABOUT ME, RESEARCH (with sub-links for OVERVIEW, SILICATE GLASSES, CELLULOSE NANOFIBRILS, and DIFFUSE FRACTURE MODELING), PUBLICATIONS, CONTACT, and LINKS. On the left, there is a 3D visualization of a specimen with dimensions: height 5.0, width 2.0, and depth 5.0. To the right of the visualization is a diagram of a simple tension test setup with a red rectangular element labeled "Diffu". A blue banner at the bottom of the page contains the text "FORTRAN files", "INPUT files", and "Tutorials".

Diffu

1. Simple tension with 2 elements

The tutorial presents a simple conversion between the input file generated by ABAQUS and the use of the new UEL.

The instructions can be downloaded from [here](#). While the files used and created through the tutorial are accessible from [this link](#).

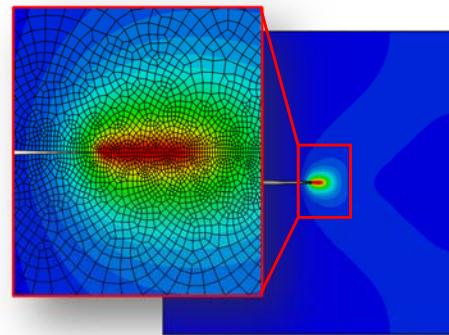
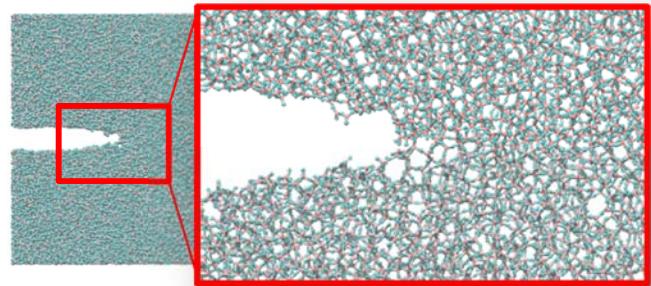
$du = 0.1$

$2 \times 0.50 = 1.00$

1.00

FORTRAN files
INPUT files
Tutorials

Conclusion



XFEM/GFEM
Cohesive Zones

Phase-field

Predefined
crack

Advantages and disadvantages

- crack initiation, propagation
- branching, merging
- fixed mesh
- fully 3D



- fine mesh
- finite crack size
- efficiency/robustness

Versatility

dynamics, shells, nonlinear elasticity, large strains,
coupled problems, plasticity, anisotropy, etc...

References

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- D. B. Mumford, J. Shah. , Comm. Pure Appl. Math. 42(5) (1989) 577-685.

Thank you for your attention

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