LaMCoS
Laboratoire de Mécanique
des Contacts et des Structures
UMR 5259

Fracture toughness of periodic beam lattices

Gergely Molnár, Julien Réthoré

Journées d'automne du GDR ARCHI-META, Lille

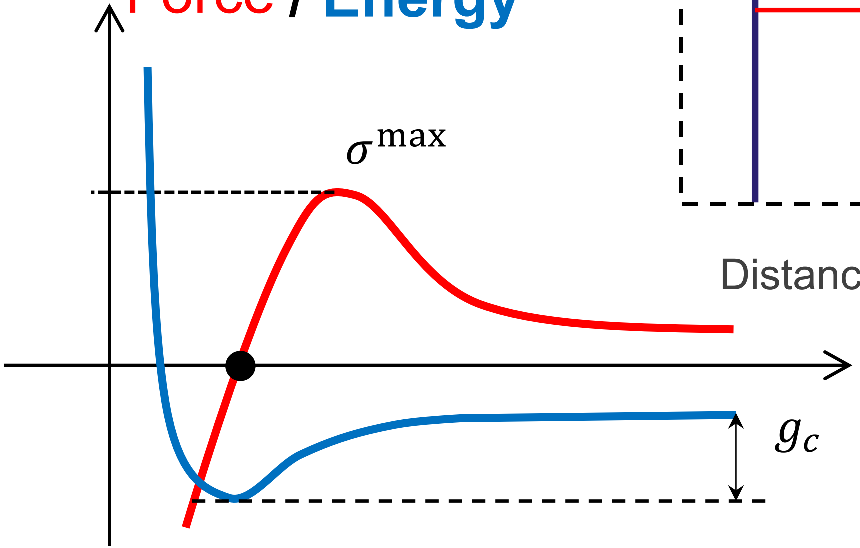
29/11/2024

cnrs
INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON
UNIVERSITÉ DE LYON
INSTITUT CARNOT Ingénierie@Lyon

Motivation

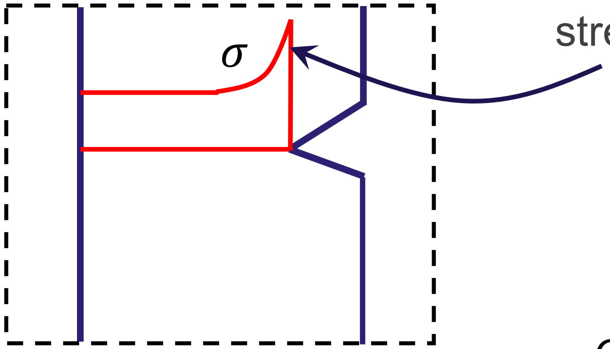
Energy release rate

Force / **Energy**

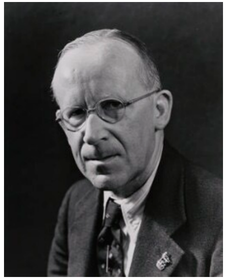


typical interatomic potential

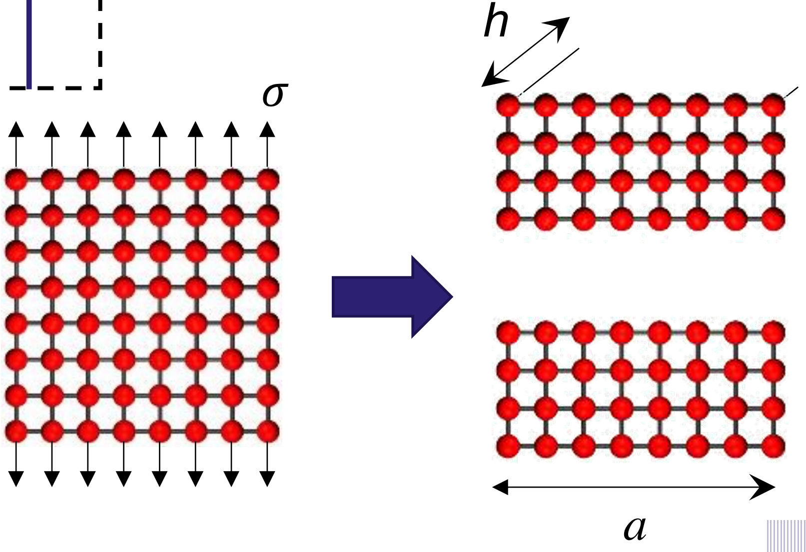
$$S = 2 g_c h a$$



stress singularity

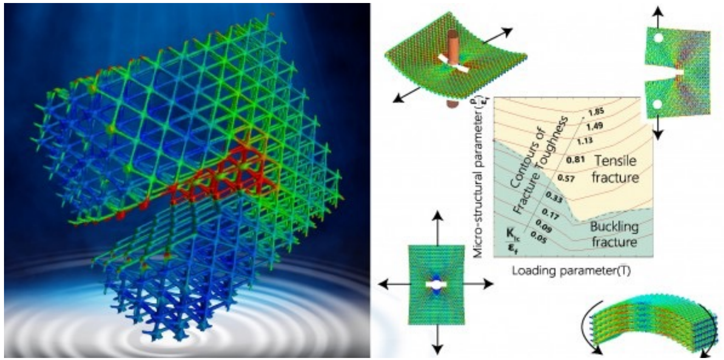


Griffith (1921,1924)



Motivation

Toughness of metamaterials

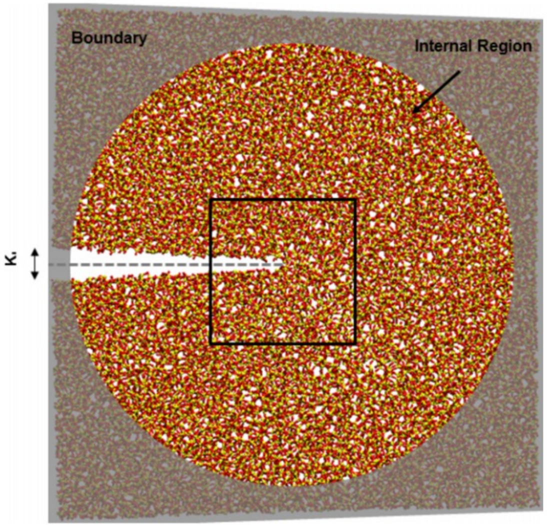


Shaikeea et al. (2022), Nature Materials

Asymptotic analysis:

$$\tilde{\sigma}_{ij} \equiv \frac{\sigma_{ij}}{\sigma_f} = \bar{K}_{\mathrm{I}}\left[\bar{r}^{-\frac{1}{2}} f_{ij}^{I} + \bar{T}_{11}\delta_{1i}\delta_{1j} + \bar{T}_{33}\delta_{3i}\delta_{3j} + \bar{T}_{13}\delta_{1i}\delta_{3j} + \mathcal{O}(\bar{r}^{\frac{1}{2}}) + \mathcal{O}(\bar{r}) + \ldots\right]$$

Toughness of amorphous matter

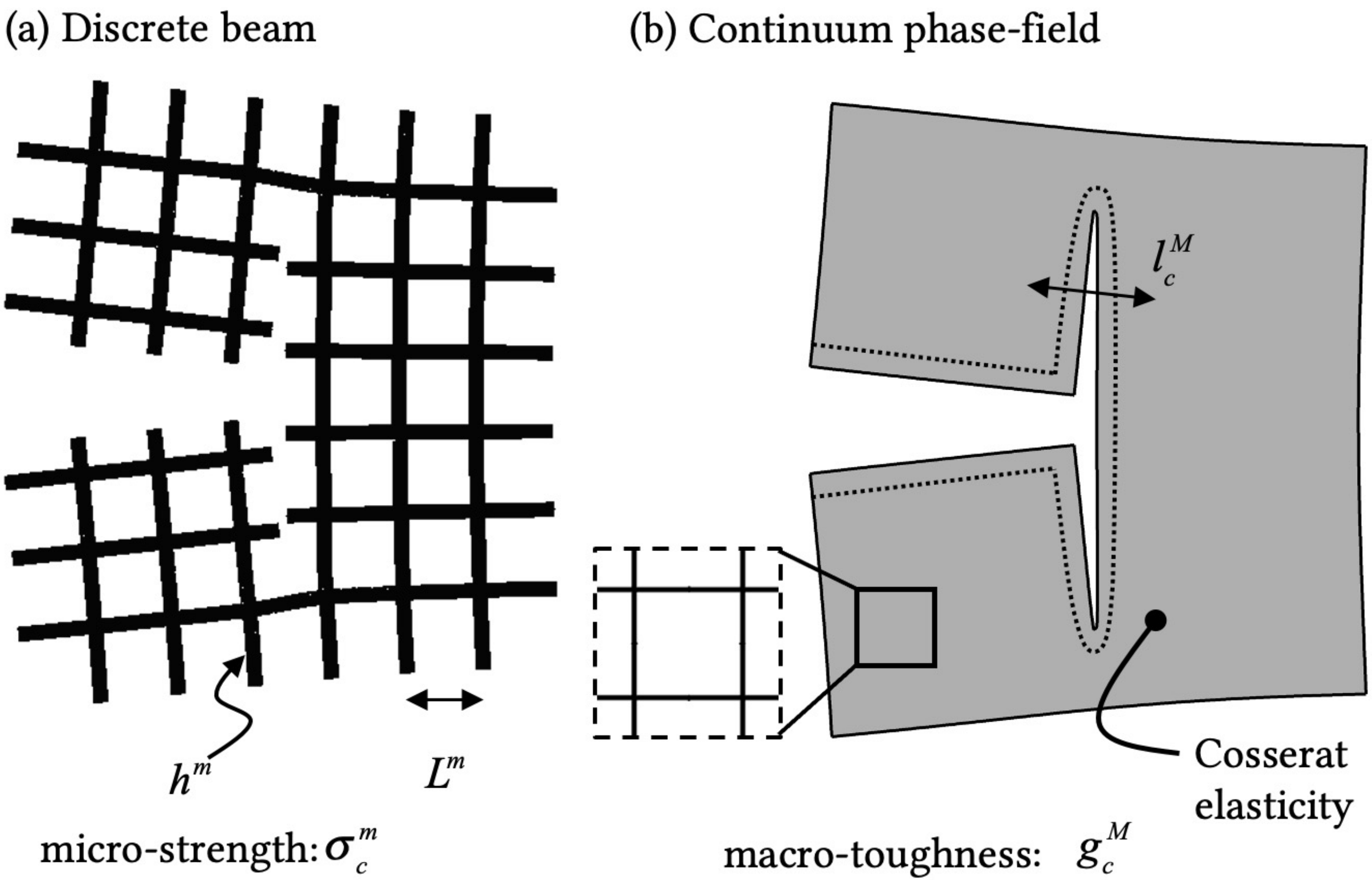


J integral

$$J_I = \int_S \left(\Pi_\varepsilon n_x - \mathbf{t}\frac{\partial \mathbf{u}}{\partial x} \right) dS$$

QUESTION:
Can we calculate toughness based on an RVE?

Problem statement



- Critical load
- Fracture topology
- Various loading

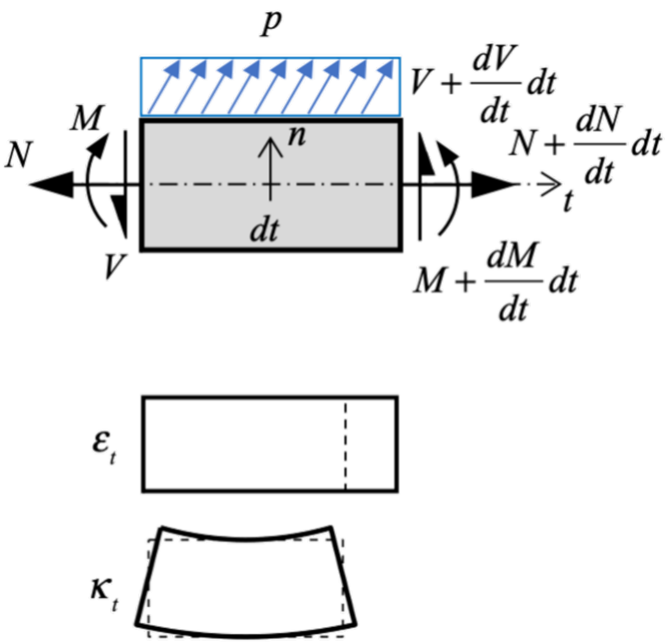
Beam model

Euler-Bernoulli beams

<u>Modelling assumptions</u>

- beams are flawless
- no stress

  concentration at joints
- $\sigma_t < \sigma_c^m$

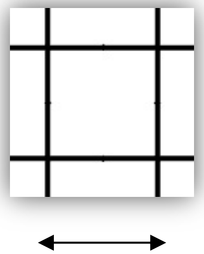


Nguyen et al. (2017)

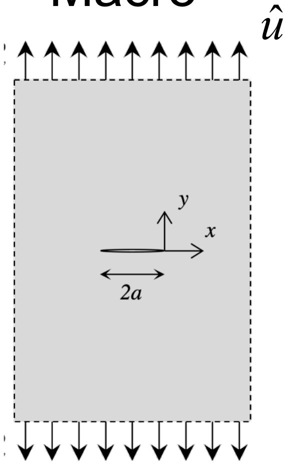
Beam model

Toughness calculation

Micro



Macro



L^m

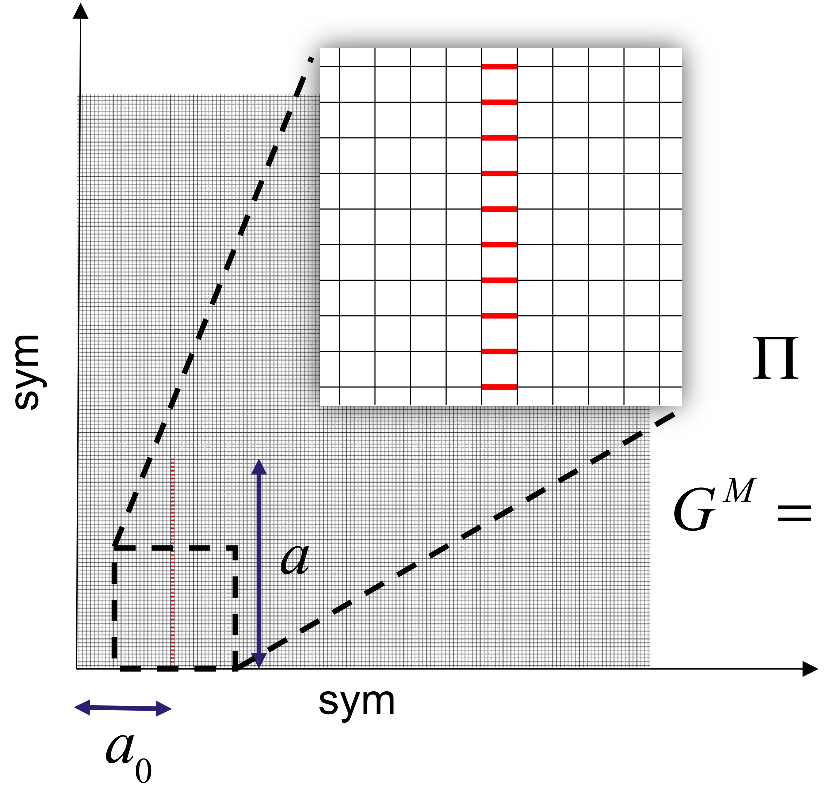
h^m

σ_c^m

?

σ_c^M

$g_c^M \approx G^M$

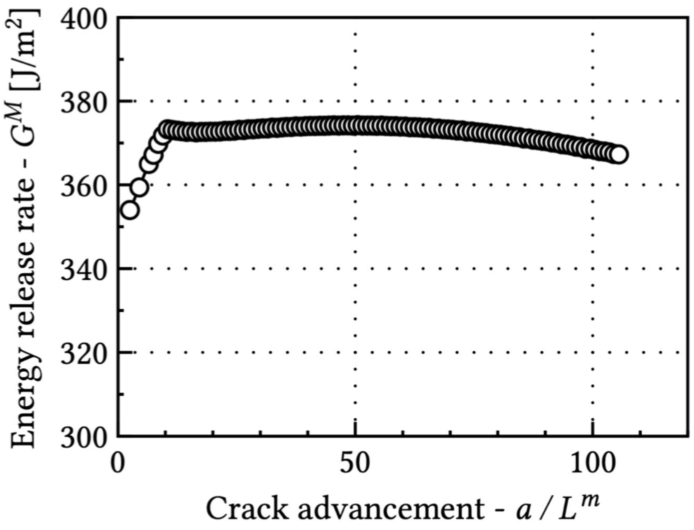


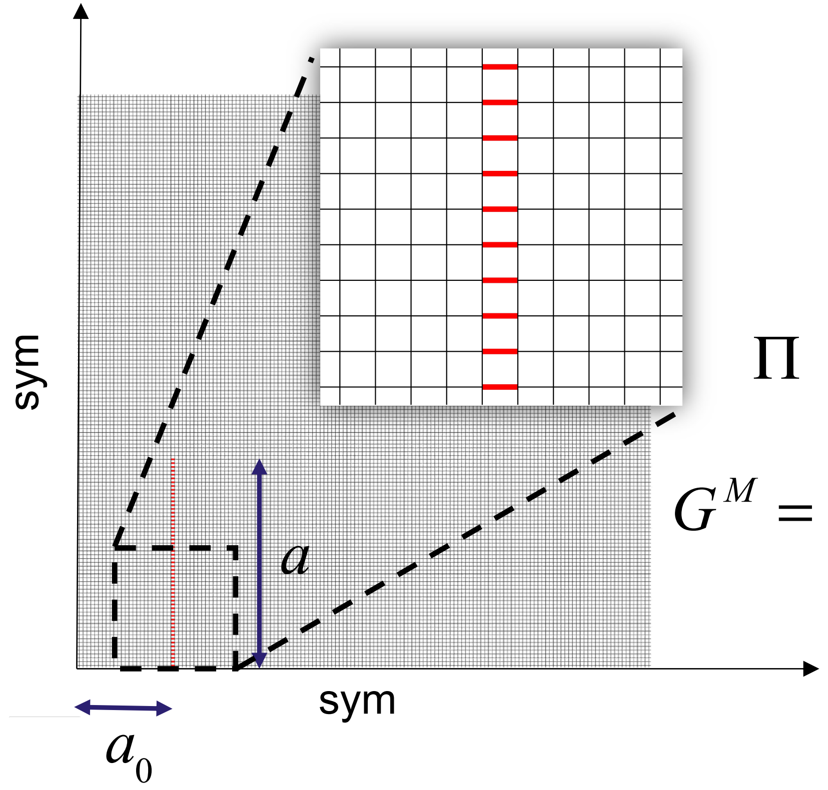
Π potential energy

$$G^M = -\frac{d\Pi - W_{\text{ext}}}{da}$$

Beam results

Toughness calculation



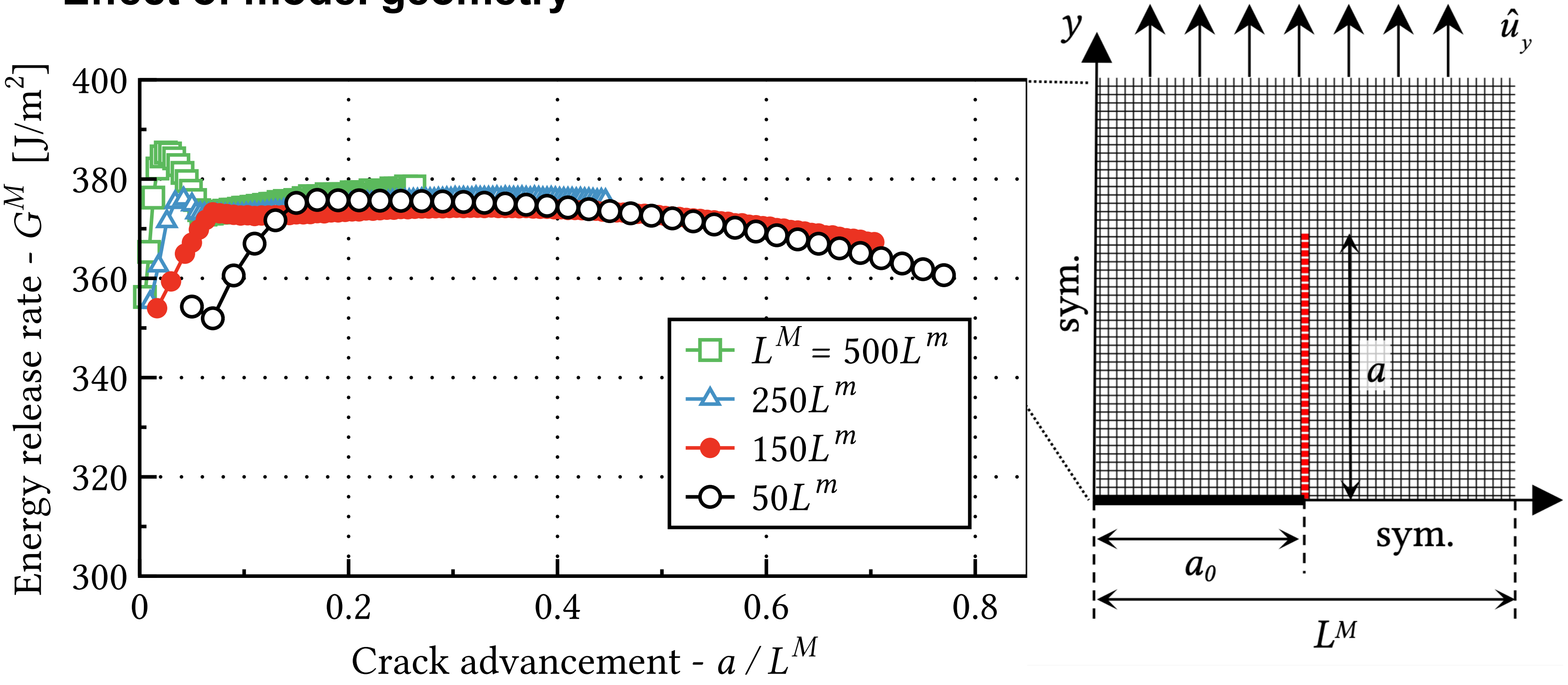


Π potential energy

$$G^M = -\frac{d\Pi - W_{ext}}{da}$$

Beam results

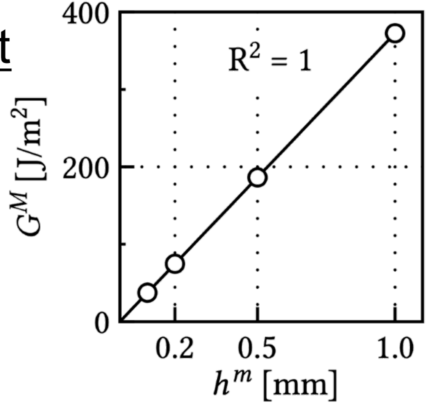
Effect of model geometry



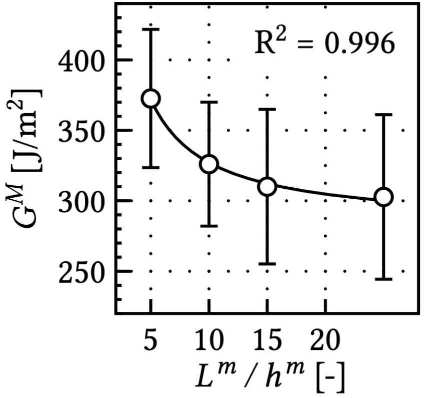
Beam results

Effect of microstructure and material

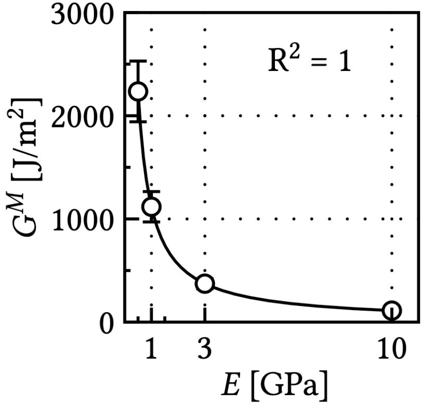
Beam height



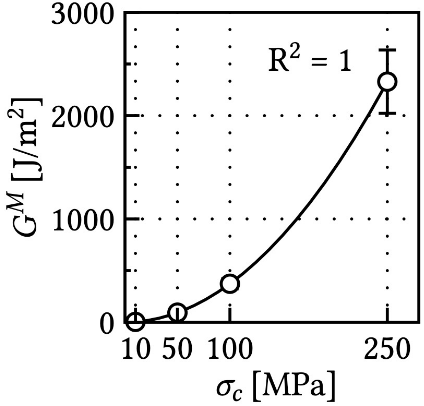
Slenderness



Young's modulus



Strength



$$G^M \sim g_{\text{Irwin}} \approx \frac{\left(\sigma_c^m\right)^2 h^m}{E}$$

Question

Fracture toughness based on RVE?

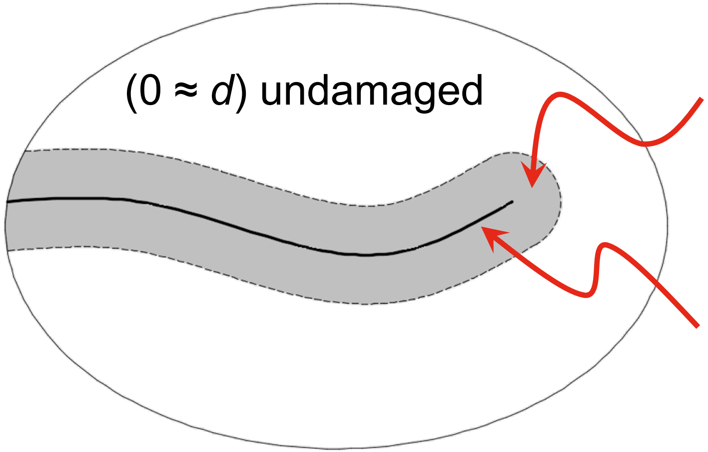
Continuum model

Phase-field theory

1. Brittle fracture $$-\frac{\partial \psi}{\partial a} = \frac{\partial S}{\partial a} = g_c$$ (Griffith, 1920)

crack energy density

2. Minimization problem $$\Pi(\mathbf{u}, d) = \int_\Omega g(d)\psi_0(\varepsilon(\mathbf{u}))\,d\Omega + \int_\Omega \frac{3g_{c,M}}{8l_c}\left(d + l_c^2 |\nabla d|^2\right) d\Omega$$



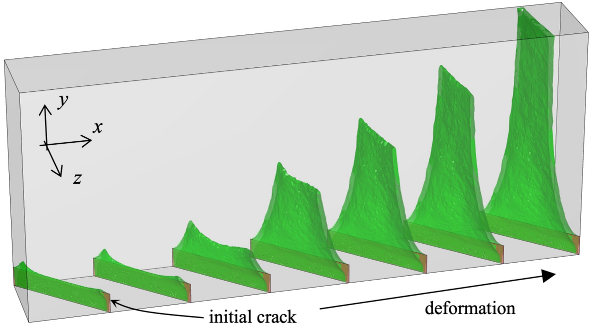
(Ambrosio & Tortorelli, 1990)
(Bourdin et al., 2000)
(Amor et al., 2009)
(Miehe et al., 2010)

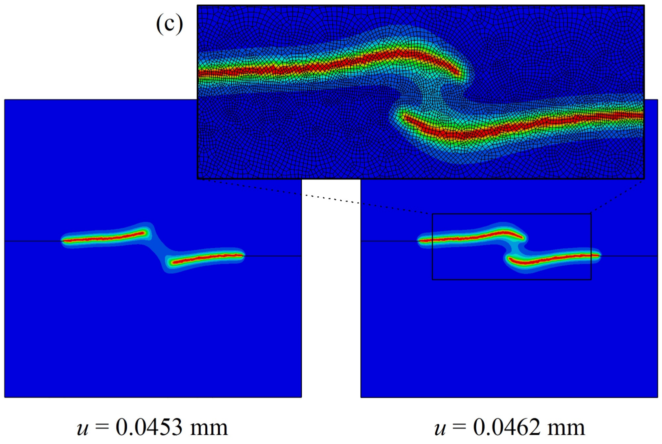
$l_c \to 0 \quad \Gamma$ converges

Continuum model

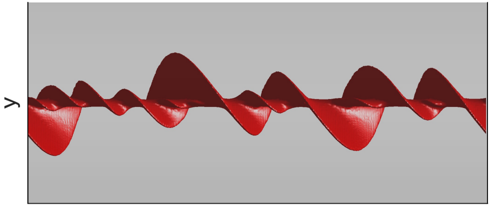
Molnár & Gravouil (2017)

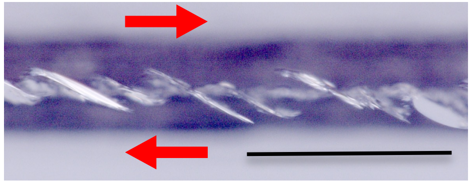
Phase-field examples

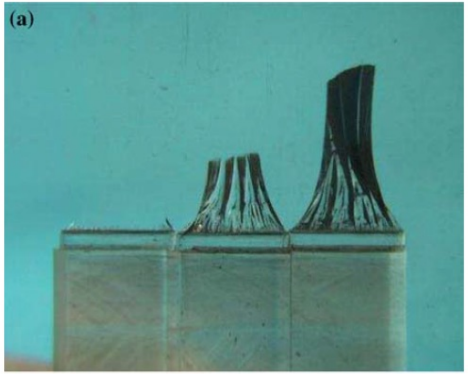




Molnár et al. (2024)







Mode I+III

Lazarus et al. (2008)

Mode I+II

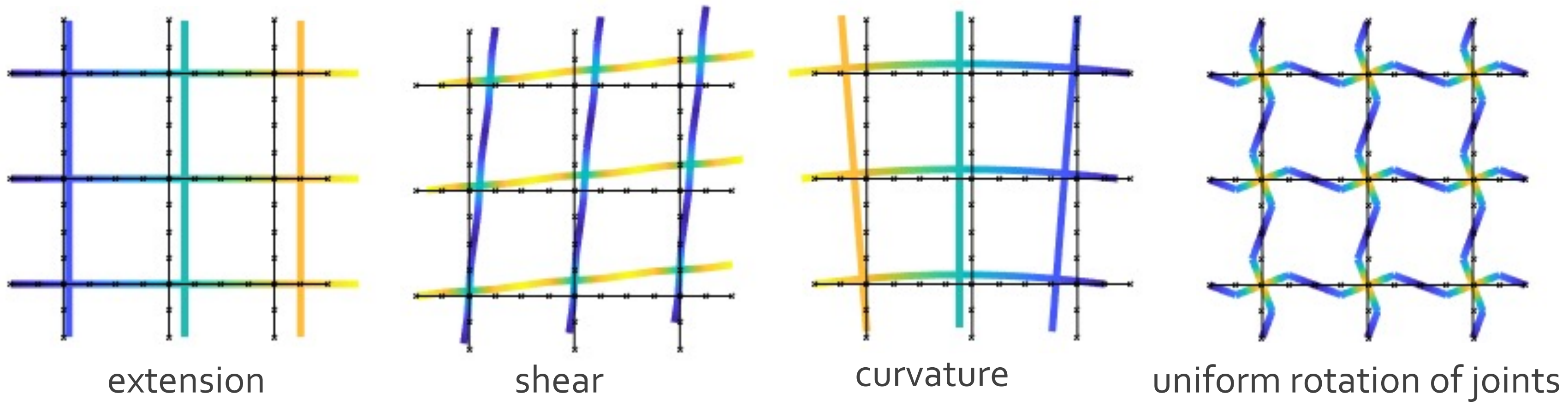
Continuum model

Cosserat theory

New DOF: rotation ϕ

$$\Pi(\mathbf{u},\phi,d)=\int_{\Omega} g(d)\cdot\ \psi(\varepsilon(\mathbf{u},\phi))\ d\Omega+W(d,\nabla d)$$

New deformations

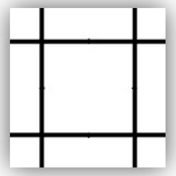


(Cosserat & Cosserat, 1909)

Continuum model

Bleyer & Alessi (2018)
Scherer et al. (2022)

Multiple damage fields



$$\Pi\left(\mathbf{u},d\right)=\Psi\left(\mathbf{u},\phi,g_{ij}\left(d_i\right)\right)+\sum_{i=1}^{2}\frac{3g_{c,M,i}}{8l_{c,M,i}}\int_\Omega\left(d_i+l_{c,M,i}^2\left|\nabla d_i\right|^2\right)d\Omega$$

Degradation functions

$$\Psi\left(\mathbf{u},\phi,g_{ij}\left(d_i\right)\right)=\frac{1}{2}\int_\Omega \underline{\varepsilon}^T \underline{\underline{C}}\left(g_{ij}\left(d_i\right)\right)\underline{\varepsilon}\, d\Omega$$

$$g_{ij}\left(d\right)=\left(\frac{1-d}{1+d\gamma_{ij}}\right)^2$$

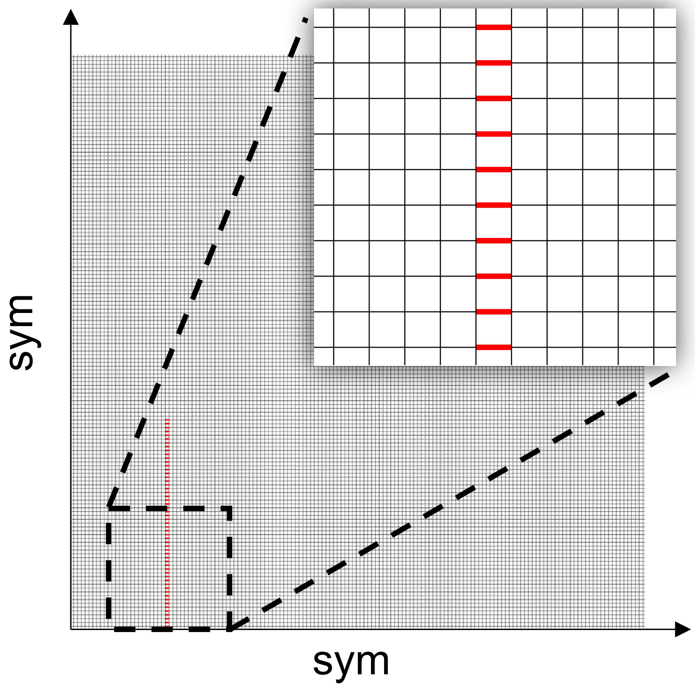
Lorentz & Godard (2011)

$$\underline{\underline{C}}=\begin{bmatrix} g\left(d_1,\gamma_{11}\right)C_{11} & & & & & \\ & g\left(d_2,\gamma_{22}\right)C_{22} & & & \varnothing & \\ & & g\left(d_2,\gamma_{23}\right)C_{33} & & & \\ & & & g\left(d_1,\gamma_{14}\right)C_{44} & & \\ & sym. & & & g\left(d_1,\gamma_{55}\right)C_{55} & \\ & & & & & g\left(d_2,\gamma_{66}\right)C_{66} \end{bmatrix}$$

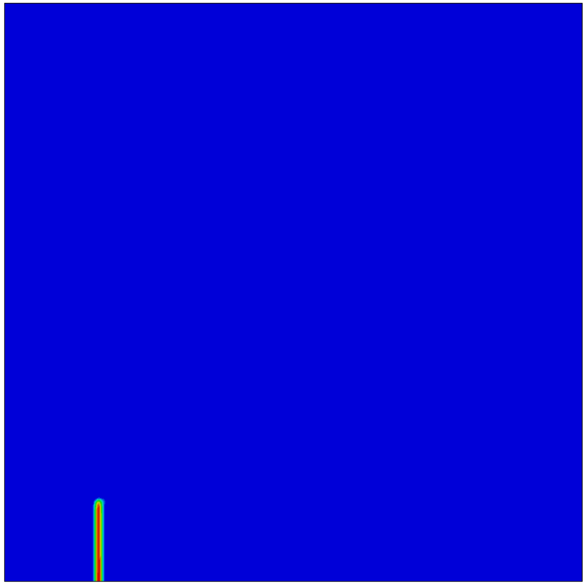
Calibration

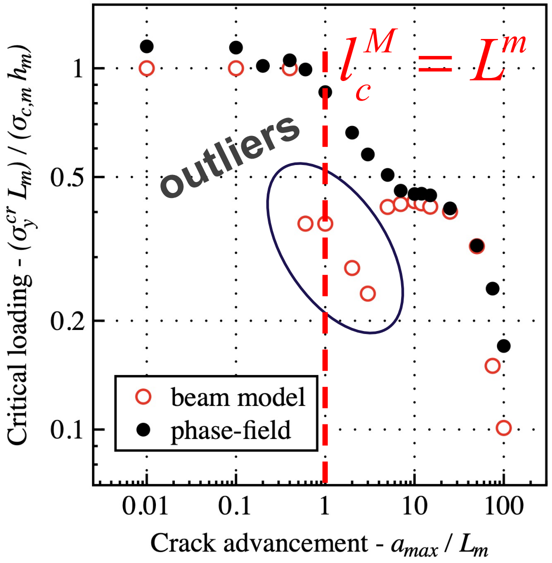
Square grid in tension (Mode I)

Beam model



Phase-field

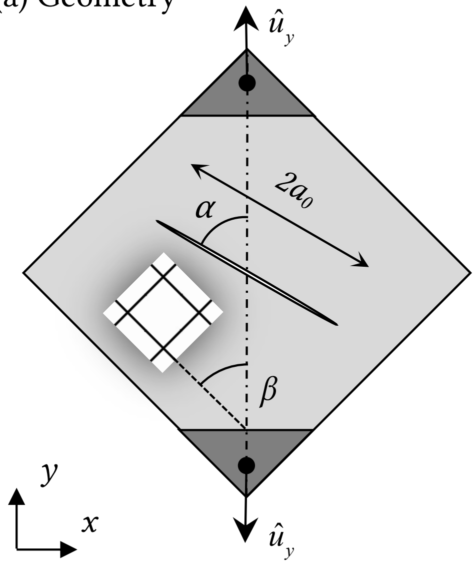




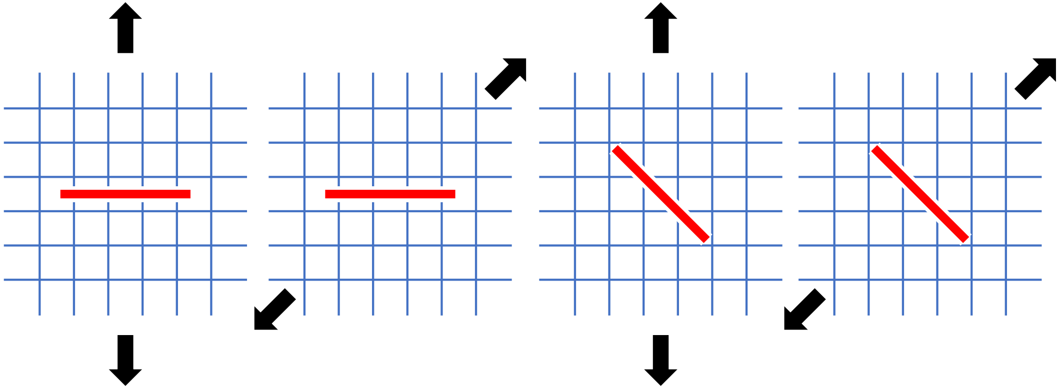
Experimental validation

Principal orientations

(a) Geometry



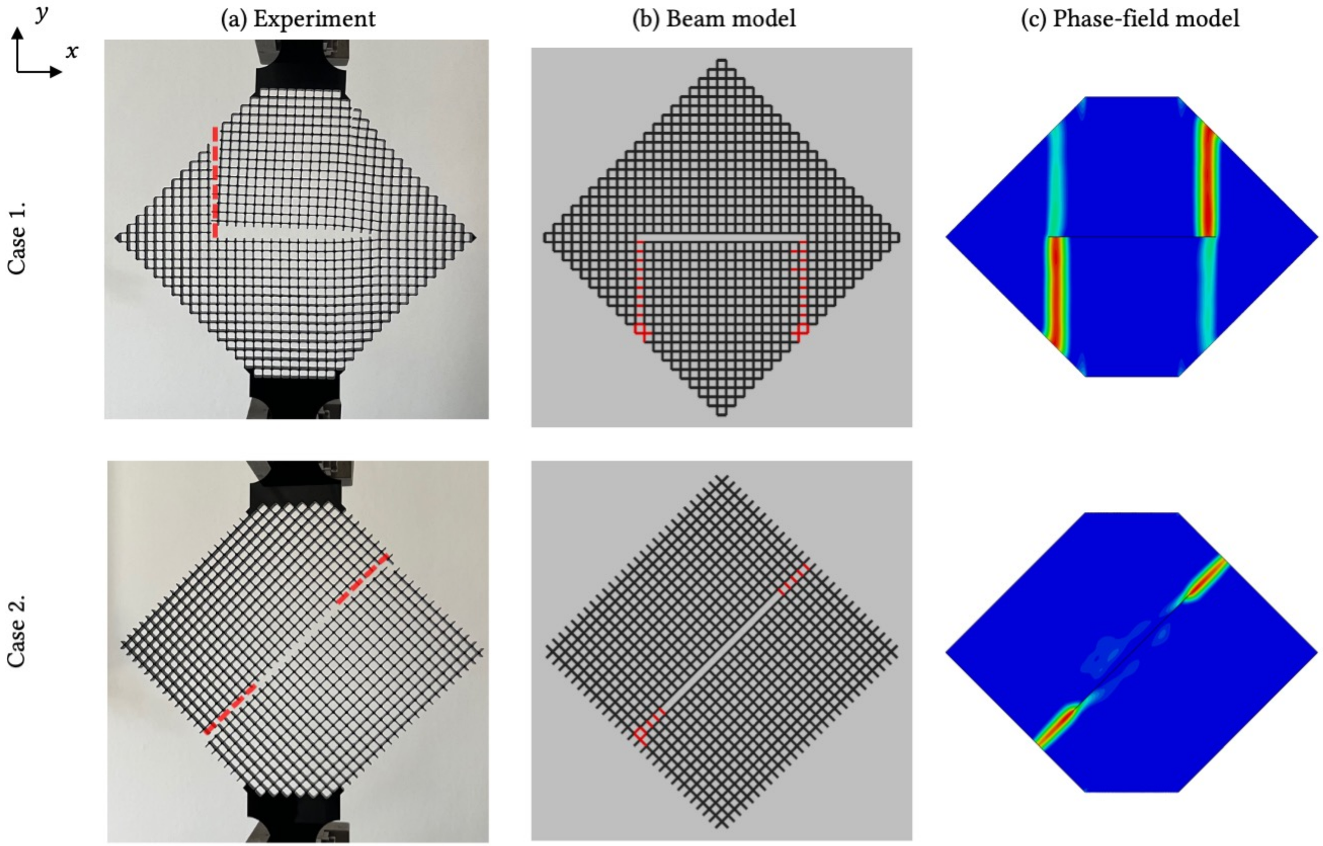
(b) Orientations

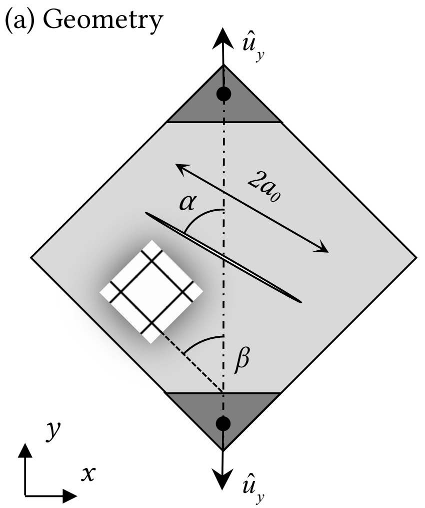


	Case 1	Case 2	Case 3	Case 4
Crack/Loading (α):	90°	45°	45°	90°
Structure/Loading (β):	90°	45°	90°	45°
Crack/Structure :	0°	0°	45°	45°

Experimental validation

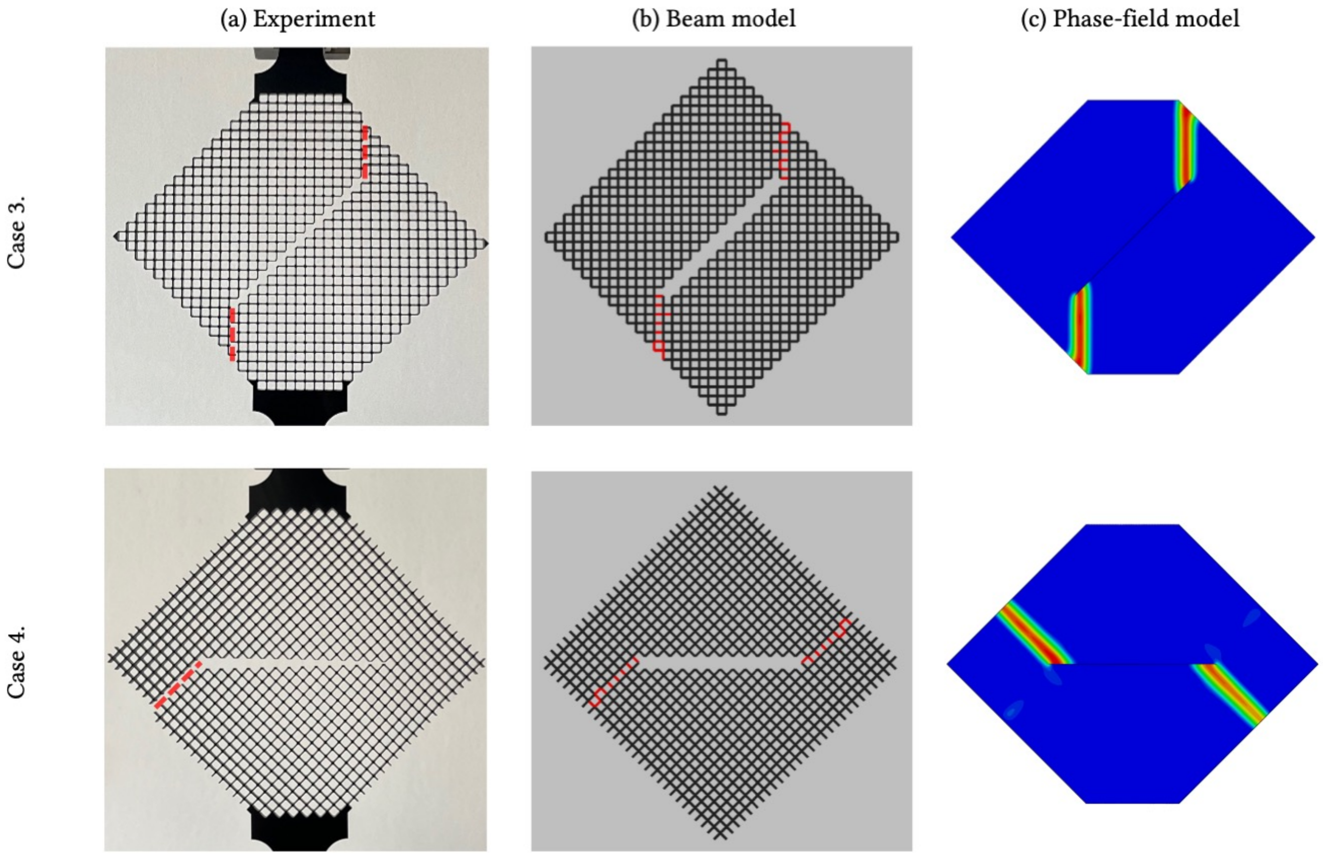
Tension and shear

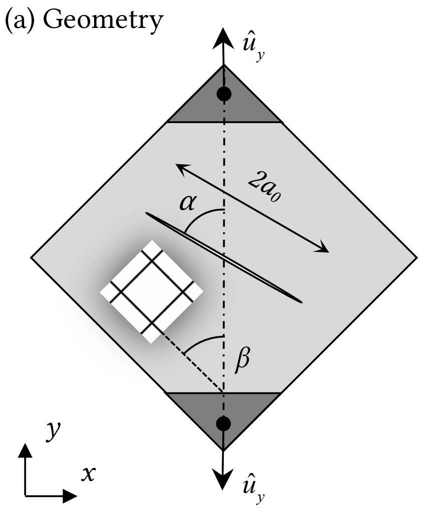




Experimental validation

Tension and shear



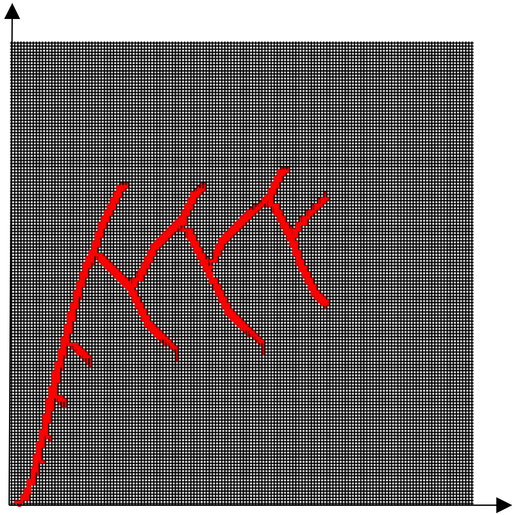


Conclusion / Perspectives

Fracture in periodic beam lattices

- **Toughness** can be defined
- Follows **continuum** theories

$$G^M \sim g_{\mathrm{Irwin}} \approx \frac{\left(\sigma_c^m\right)^2 h^m}{E}$$



- **Homogenization**
  - Cosserat continuum
  - Multiple damage variables
  - Anisotropic phase-field

→ **Other lattice types, 3D, etc…**

Thank you for your attention

Try it out!

G. Molnár et al., Thermodynamically consistent linear-gradient damage model in Abaqus, **Engineering Fracture Mechanics**, 108390, 2022.

G. Molnár & J. Réthoré, Fracture Toughness of Periodic Beam Lattices**, Journal of Theoretical, Computational and Applied Mechanics** (submitted, 2024) hal-04793587

Please visit

www.molnar-research.com

gergely.molnar@insa-lyon.fr